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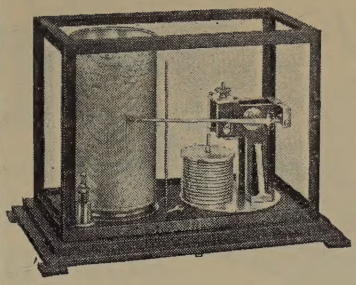

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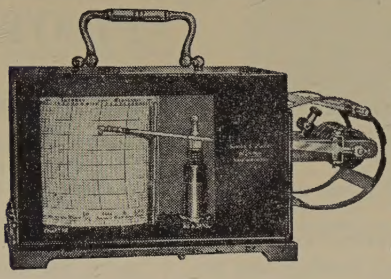

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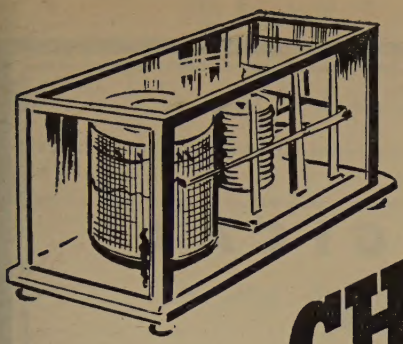

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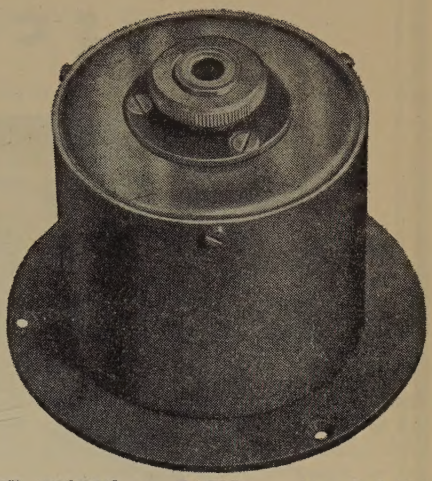

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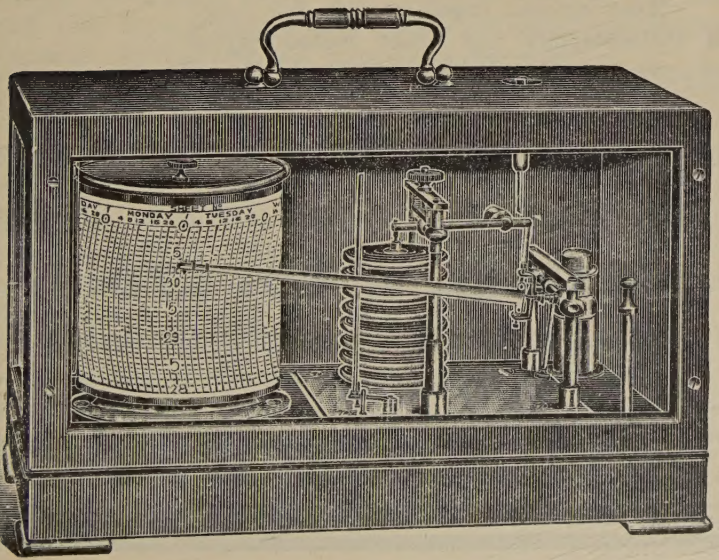

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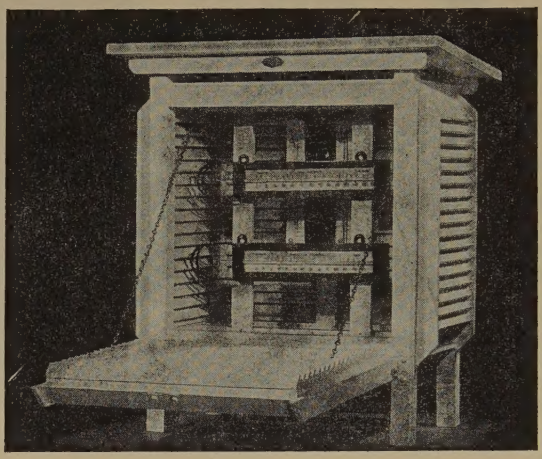

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On the dynamics of the general circulation

By ROBERT G. FLEAGLE
University of Washington, Seattle, U.S.A.

(Manuscript received 9 July 1956)

SUMMARY

The dominant features of large-scale atmospheric motion emerge from a relatively simple treatment of the hydrodynamic equations, thus providing clear, intuitive understanding of the relation between cause and effect in the general circulation. The analysis is made possible by applying the perturbation equations to a sloping stream surface located in the middle troposphere. For reasonable values of the arbitrary parameters, the results show that baroclinic disturbances are unstable poleward of a critical latitude, roughly 35°, and that the preferred number of such unstable disturbances at any one time decreases from about 10 at 45° to 2 at 80°. Greatest intensity occurs at about 60°. Jet streams must form close to the critical latitude and, in the case of long disturbances, at higher latitudes. Anticyclonic shear has a stabilizing influence, so that the normal velocity distribution in middle latitudes offers considerable resistance to penetration of disturbances to low latitudes. Below the critical latitude unstable disturbances are possible only in relatively small areas where the hydrostatic stability is nearly zero. Growth of these disturbances is favoured by anticyclonic shear, as well as by baroclinicity and by high latitude. Disturbances of maximum rate of growth are practically non-dispersive so that their growth depends mostly on available potential energy. If sufficient energy is available, the identity of individual disturbances can be maintained over extended time periods. Wave speed relative to the zonal current exhibits a maximum positive value at a high latitude.

1. INTRODUCTION

During the past score of years steady increase in the quantity and in the accuracy of upper-air data has brought description of the large-scale motion of the atmosphere into sharper and sharper focus. The picture provided by these data, although complex in detail, may be organized around certain persistent and recurrent features. Among these features the following are particularly dominant :

1. In middle and high latitudes the atmosphere is subject to large-amplitude unstable disturbances, whereas in low latitudes the flow is for the most part relatively slow and steady. However, unstable disturbances of smaller scale, though great intensity, develop at low latitudes.

2. Individual middle and high-latitude disturbances extend through a wide latitude belt, others are limited in latitudinal extent. The number of disturbances at any one time in winter decreases with latitude from something like eight or ten at 45° lat. to two or three in polar regions.

3. The more intense disturbances develop into vortices, and the intensity of such vortices is greatest at about 60° lat., decreasing toward the north and south.

4. A high-speed zonal current (jet stream) often is present in middle latitudes in the upper troposphere and sometimes is present at higher latitudes.

It is to be noted that these are properties of specific flow patterns and are not necessarily properties of a hypothetical flow pattern obtained by averaging over time or longitude.

The features listed above will be considered to define broadly the term, general circulation. The definition could be stated in various more or less equivalent ways, and many other features could be listed. In particular, certain consequences of the features listed above, such as latitudinal energy transfer, could be included as features of the general circulation. Indeed, it is characteristic of the subject of the general circulation that there are nearly as many definitions of the term as there are meteorologists interested in the subject. This condition reflects the fact that thorough understanding of the cause of the general circulation does not exist. Mysteries are always hard to define, and capable observers give varying reports when confronted by phenomena they do not understand.

There is, of course, complete agreement that the fundamental cause of the general circulation, as well as of virtually all except tidal atmospheric motions, is differential heating by the sun. This has been recognized at least since Hadley (1735-6). Beyond this, there is general agreement that the earth's rotation provides an essential part of the mechanism, but just how this mechanism operates has not been clearly stated. So the theory of the general circulation has been developed only to the most rudimentary stages; it is an instructive, albeit discomforting, experience to be queried by non-meteorologists about the cause of these central meteorological phenomena and to realize that an intelligent layman can answer such questions almost as well as a highly-trained meteorologist.

The problem may be recognized, therefore, as one of explaining the features listed earlier; the explanation should be based on fundamental physical principles and expressed in the physical scientist's language, mathematics. A complete solution should emanate from integration of the hydrodynamic equations including expressions of the physical processes associated with phase change, radiation, and turbulent diffusion. In this complete solution, solar radiation incident on the earth's atmosphere and the physical characteristics of the earth's surface would be specified, but the response of the atmosphere to these conditions would be a product of the integration. This is an ambitious goal which is far beyond present capabilities, but it is the only complete solution to the problem.

A less ambitious problem may be imagined in which the hydrodynamic equations applied to an inviscid, polytropic atmosphere are linearized and periodic disturbances in time and longitude are assumed. The latitudinal and vertical dependence would be unspecified, and this would lead to very severe mathematical difficulties. If either of these were specified, it might be possible to develop an ordinary differential equation with variable coefficients which, in principle, is soluble. For certain simplified models of the atmosphere, in particular, adiabatic inviscid flow and no horizontal shear of the undisturbed wind, Taylor (1936), Pekeris (1937), Charney (1947), Eady (1949), Kuo (1952), and others have successfully solved problems of this sort. These solutions have provided very great insight into the complex behaviour of the atmosphere. The studies of tidal motions by Taylor and Pekeris demonstrated the unity of the whole atmosphere and the fact that a disturbance in one region of the atmosphere inevitably influences the behaviour of the whole atmosphere. The studies of baroclinic instability by Charney, Eady, and Kuo, as well as the early study by Sutcliffe (1947), have revealed the role played by the potential energy of mass distribution in the development of cyclones and anticyclones. Briefly, these studies have shown that if the baroclinicity (horizontal temperature gradient) exceeds a certain critical value which depends on wavelength and certain parameters which characterize the atmospheric model, small quasi-horizontal disturbances will grow in amplitude. The instability represents conversion of potential energy into kinetic energy through sinking of the cold air and rising of the warm air. As a result of these studies together with the improved description of large-scale atmospheric motions, for the first time the problem of the general circulation has been rendered accessible to an

understanding of cause and effect. Eady (1950) presented one of the earliest qualitative statements of this understanding, and Bolin (1952) has given an excellent discussion of recent work and its impact on understanding of the general circulation. These two works have influenced greatly the ideas set forth in this paper.

To investigate mathematically the relation of instability of quasi-horizontal motions to the general circulation, and thereby to add certainty and detail as well as clarity to the understanding, requires relaxation of some of the conditions imposed on the models referred to earlier. Unfortunately, this is likely to lead to very great difficulty; it is easy to realize why this is so when it is recognized that the model used by Charney (1947) and Kuo (1952) required the solution of the confluent hypergeometric equation. To generalize the atmospheric model still further and thereby to raise the order of difficulty is sufficient to sober even the most courageous applied mathematicians. On the other hand, the persistence of the dominant features of the general circulation suggests a virility on the part of these features which might survive even rather rough handling of the hydrodynamic equations, so long as certain crucial processes are preserved. Observation of the general circulation together with understanding of the physics of large-scale disturbances offer hope that these crucial processes can be identified.

2. Purpose and scope

It is proposed in this paper to develop from plausible postulates mathematical results which will provide a clear understanding of those dominant features of the general circulation listed earlier. The writer believes that the main restrictions on the generality of the results stem from the following assumptions :

1. The hydrodynamic equations are linearized. This requires that the distribution of undisturbed velocity and hydrostatic stability be specified arbitrarily.
2. The motion is considered to be polytropic and inviscid.
3. The effect of conditions at one latitude on development at other latitudes is not considered.

In addition to these there is the practical limitation that superposition of solutions, though possible in principle, in practice is so tedious that clear intuitive understanding would be lost if superposition were attempted. These restrictions are sufficiently serious that specific predictions of development of cyclones, anticyclones, jet streams and the like probably cannot be made; it follows that complete solution of the problem of the general circulation cannot be claimed. The virtue of the results lies in the fact that they provide intuitive understanding of the major features of the general circulation; thereby they may serve as a stimulus and a guide in efforts directed toward a more complete treatment. It seems inevitable that complete solution to the problem of the general circulation will be possible only by numerical methods.

3. The method

The central difficulty encountered in the problem, in common with most hydrodynamic problems, lies in the freedom of the atmosphere from evident physical constraints. Non-linear partial differential equations are required to describe the atmosphere; when these equations are simplified so that solutions are possible, reasonable boundary conditions yield complex frequency equations which exhibit the greatest resistance to analysis. These considerations indicate that successful simplification may lie in the addition of reasonable mathematical constraints which, of course, must conform to the behaviour of the atmosphere. Precedent for this procedure may be drawn from the classical treatment of surface-

water waves in which the motion is constrained to be irrotational (see Lamb 1932) and from Rossby's (1939) treatment of barotropic disturbances in which the flow is constrained to be horizontal and without divergence. In common with these studies, ultimate justification for the procedure used depends upon comparison of theoretical results with observation.

By postulating a sloping stream surface, the writer has been able to express the instability criterion for baroclinic disturbances in clear and simple terms (Fleagle 1955). The requirement that the air flow adhere to this surface represents a constraint on the atmosphere; but by expressing approximately the slope by the hydrodynamic equations, it is possible to avoid introducing fictitious forces into the problem. This same simplification has been introduced in this study of the general circulation; for the sake of clarity, the argument is repeated here.

Consider a small-amplitude disturbance in a zonal current. The normal component of velocity is required to vanish at rigid impervious surfaces; therefore, just above the horizontal ground surface the streamlines are horizontal. Evidence of many studies as well as casual observations of the distribution of rainfall show that, for the most part, above the ground the streamlines slope upward toward the pole. So long as finite disturbances are considered, stream surfaces may be imagined which are defined as being everywhere tangent to the velocity vectors and which slope upward toward the pole as shown in Fig. 1. The maximum slope (δ) of the stream surfaces is reached in the troposphere, not always at the same height; higher up, the slope of the stream surfaces decreases with height. It is the crux of the argument that the stream surfaces may be postulated to form approximately spherical surfaces, of curvature very nearly that of the earth.

If the stream surface of greatest slope is selected as the θ' (colatitude), λ (longitude) surface of a spherical coordinate system, then the velocity normal to the θ', λ surface and the normal gradient of the normal velocity vanish on this surface. It follows that, if the hydrodynamic equations are applied to the stream surface of maximum slope, the vertical component of velocity and its space derivatives do not appear in the equations.

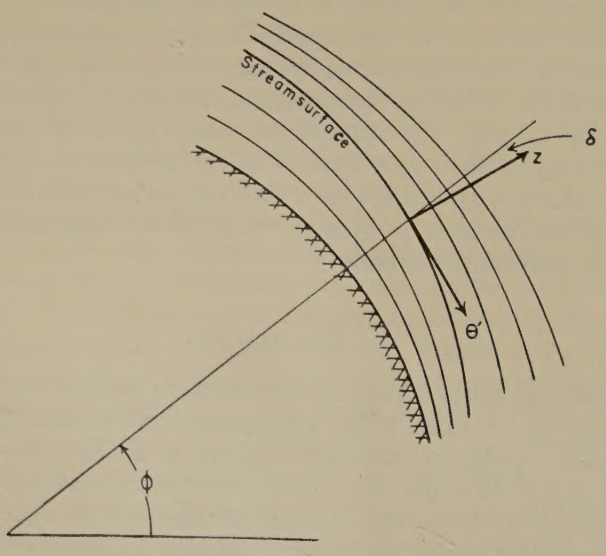

Figure 1. Vertical cross section taken along a meridian showing sloping stream surfaces, latitude angle, ϕ, and orientation of the θ', z, coordinate system.

The enormous simplification of the problem which is introduced by the elimination of the vertical velocity and its height derivative makes it possible to eliminate all five dependent variables, pressure, density, vertical pressure gradient, and the velocity components in the λ and θ' directions without obtaining the perturbation solutions explicitly. This procedure, of course, leads only to the frequency equation; the vertical structure of the disturbance must be developed from the complete set of hydrodynamic equations including the vertical velocity terms. Observations indicate that the growth of disturbances occurs simultaneously and at about the same rate throughout a large part of the troposphere. Therefore, although the frequency equation derived for the stream surface of greatest slope applies directly only to that surface, it also applies by inference to a layer of considerable thickness.

4. The frequency equation

The symbols used throughout the paper are listed in the order of their use.

Table of symbols*

Eq. No.	Symbol	Definition
	θ'	colatitude measured on sloping stream surface
	λ	longitude
	z	coordinate normal to sloping stream surface
1	$\mathbf{g}$	vector acceleration of gravity
	g	scalar acceleration of gravity
	$\mathbf{i}_2, \mathbf{i}_3$	unit vectors in $-\theta'$ and z directions respectively
	δ	slope of the stream surface
2	$\boldsymbol{\omega}$	vector angular velocity of earth
	ω	scalar angular velocity of earth
3	U	undisturbed velocity in λ direction
	a	radius of earth
	K	angular velocity of air about earth's axis relative to earth's surface
	ϕ	latitude
4	P	undisturbed pressure
	Q	undisturbed density
	θ	colatitude
5	l	$2(\omega + K)\cos\theta$
	t	time
6	$\mathfrak{L}$	$\partial/\partial t + (U/a\sin\theta)\,\partial/\partial\lambda$
	r	radial coordinate of spherical system fixed at centre of earth
	u	perturbation velocity component in λ direction
	v	perturbation velocity component in $-\theta$ and $-\theta'$ directions
	w	perturbation velocity component in r direction
	p	perturbation pressure
7	q	perturbation density
10	γ	coefficient of piezotropy
12	A	amplitude of u perturbation
	B	amplitude of v perturbation
	C	amplitude of w perturbation
	D	amplitude of p perturbation
	E	amplitude of q perturbation

* Plain subscripts, except as noted, represent partial differentiation with respect to the indicated variable. Subscripts in brackets indicate value at point indicated by subscript. Bar over symbol denotes mean value.

Eq. No.	Symbol	Definition
12	m	zonal (longitudinal) wave number
	ν	meridional (latitudinal) wave number
	σ	complex angular frequency
14	$\varkappa$	$\sigma - m K$
15	α^2	$[(m^2 - \cos^2\theta)/\sin^2\theta + \nu^2]/a^2$
17	s_z	$-(Q_z/Q + g\gamma)$
29	σ_R	real part of σ for unstable disturbances
30	σ_i	imaginary part of σ
32	ϕ_c	critical latitude
35	h	height of stream surface of maximum slope
39	c_R	speed of unstable disturbance
40	G	group velocity
	L	wavelength

In the sloping coordinate system described above $\mathbf{g}$ and $\boldsymbol{\omega}$ may be resolved into components as follows :

$$\mathbf{g} = -g(\mathbf{i}_2 \sin\delta + \mathbf{i}_3 \cos\delta) \quad . \quad . \quad . \quad . \quad (1)$$

$$\boldsymbol{\omega} = \omega[\mathbf{i}_2 \cos(\phi - \delta) + \mathbf{i}_3 \sin(\phi - \delta)] \quad . \quad . \quad (2)$$

The perturbation method as described by Haurwitz (1951) provides a straightforward method for linearizing the hydrodynamic equations and therefore is the basis for the mathematical development of this paper. The undisturbed motion will be assumed to be characterized by zonal flow (U) which increases linearly with height and whose angular velocity is a constant (K). The dependence of the zonal speed on latitude is given by

$$U = a K \cos\phi \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The equations of undisturbed motion then become

$$\left.\begin{aligned} U[2\omega\cos\theta' + (U/a)\cot\theta'] &= P_{\theta'}/aQ - g\sin\delta \\ g\cos\delta &= -Q^{-1}P_z \end{aligned}\right\} \quad . \quad . \quad (4)$$

If δ is considered small, $\sin\delta$ may be approximated by δ and $\cos\delta$ by unity. Elimination of P from Eq. (4) then gives to an excellent approximation

$$\frac{gQ_{\theta'}}{aQ} = -U_z l - \frac{Q_z}{Q}[g\delta + U(l - K\cos\theta)] \quad . \quad . \quad . \quad (5)$$

where the small difference between $\cos\theta$ and $\cos\theta'$ or between $\sin\theta$ and $\sin\theta'$ is neglected. In the case of a non-spherical stream surface, the right-hand side of Eq. (5) should contain the term $g\delta\delta_{\theta'}/a$; curvature of the stream surface such that δ changes by 100 per cent in one radian gives for this term a magnitude four or five orders smaller than the dominant terms.

The perturbation equations may be written in spherical coordinates for the case of quasi-hydrostatic equilibrium in the form

$$\mathcal{L}(u) + (v/a)U_\theta + \underline{wU_r} - lv = -(Qa\sin\theta)^{-1}p_\lambda \quad . \quad . \quad (6)$$

$$\mathcal{L}(v) + lu = (Qa)^{-1}p_\theta - qQ^{-2}a^{-1}P_\theta \quad . \quad . \quad . \quad (7)$$

$$gq\cos\delta + p_r = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$\left.\begin{aligned} \mathcal{L}(q) - (v/a)Q_\theta - Q(v/a)\cot\theta + Qu_\lambda/a\sin\theta \\ -(Q/a)v_\theta + (\underline{Qw})_r = 0 \end{aligned}\right\} \quad . \quad . \quad (9)$$

$$\mathcal{L}(q) - (v/a)(Q_\theta - \gamma P_\theta) + w(\underline{Q_r - \gamma P_r}) - \gamma\mathcal{L}(p) = 0 \quad . \quad (10)$$

where v is positive toward the north. Elimination of p_θ between Eqs. (6) and (7) gives

$$\left.\begin{aligned}&\mathcal{L}(v_\lambda + u_\theta \sin\theta) + l(u_\lambda - v_\theta \sin\theta) + v(2\omega \sin^2\theta + K\sin^2\theta\\&+ \underline{w_\theta U_r \sin\theta} + q_\lambda P_\theta/aQ^2 - p_\lambda (Q_\theta/Q + \cot\theta)/aQ = 0\end{aligned}\right\} \quad . \quad (11)$$

These equations may be used in the usual λ, θ, r spherical coordinate system or, by omitting the underlined terms and replacing r by z and θ by θ', they may be used in the sloping spherical system.

Solutions which are periodic in longitude, latitude and time may be written

$$\left.\begin{aligned}u &= A(r) \exp i[m\lambda + \nu\theta - \sigma t]\\v &= B(r) \exp i[m\lambda + \nu\theta - \sigma t]\\w &= C(r) \exp i[m\lambda + \nu\theta - \sigma t]\\p &= D(r) \exp i[m\lambda + \nu\theta - \sigma t]\\q &= E(r) \exp i[m\lambda + \nu\theta - \sigma t]\end{aligned}\right\} \quad . \quad . \quad (12)$$

Upon applying these solutions to the sloping surface, $C(r)$ vanishes, r is replaced by z and θ is replaced by θ'. Then, substitution of Eq. (12) in Eqs. (6), (8), (9), (10) and Eq. (11) at the stream surface of greatest slope yields a set of five simultaneous algebraic equations in A, B, D, D_z, and E. In order that non-trivial solutions exist the determinant of the coefficients must vanish. This condition yields the following quartic equation:

$$\left.\begin{aligned}&-\gamma m x^4 + \gamma x^3 \sin\theta' [K\sin\theta' + i\nu K\cos\theta' + l_{\theta'}]\\&+ m x^2 \left[\alpha^2 - \frac{i\nu}{a^2 Q}(Q_{\theta'} + 2Q\cot\theta') + \underline{\gamma l(l - K\cos\theta')}\right]\\&- \frac{m^2 x}{a^2 \sin\theta'}\left[\frac{K\cos\theta'}{Q}(\gamma P_{\theta'} - Q_{\theta'} - Q\cot\theta') + l_{\theta'}\right.\\&\left.+ \frac{l}{Q}(Q_{\theta'} - 2\gamma P_{\theta'}) + \underline{K\sin\theta'}\right] - \frac{m^3 P_{\theta'}(Q_{\theta'} - \gamma P_{\theta'})}{a^4 Q^2 \sin^2\theta'} = 0\end{aligned}\right\} \quad . \quad (13)$$

where

$$x \equiv \sigma - mK \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

$$\alpha^2 \equiv \frac{1}{a^2}\left(\frac{m^2 - \cos^2\theta'}{\sin^2\theta'} + \nu^2\right) \quad . \quad . \quad . \quad . \quad (15)$$

The first two terms in Eq. (13) are small compared to the third for values of wave speed much less than the speed of sound. Also, for large-scale disturbances, observations show that the underlined terms are at least an order of magnitude smaller than the remaining terms. The underlined terms therefore may be neglected if care is taken to ensure that the remaining terms do not cancel at a subsequent step. Then, the following approximate equalities may be made from Eq. (4) and Eq. (5)

$$Q_{\theta'}/Q = -a\delta Q_z/Q \quad . \quad . \quad . \quad . \quad (16)$$

$$\frac{Q_{\theta'} - \gamma P_{\theta'}}{Q} = (g\delta s_z - l U_z)(a/g) \quad . \quad . \quad . \quad (17)$$

$$\frac{Q_{\theta'} - 2\gamma P_{\theta'}}{Q} = -ag\delta\gamma \quad . \quad . \quad . \quad . \quad (18)$$

Substitution of Eqs. (16), (17), and Eq. (18) in Eq. (13) gives

$$\left.\begin{aligned} & x^2\left[\frac{m^2-\cos^2\theta'}{\sin^2\theta'}+\nu^2+i\,\nu\,(a\,\delta\,Q_z/Q-2\cot\theta')\right] \\ & -\frac{x\,m}{\sin\theta'}(l_{\theta'}-a\,g\,\gamma\,l\,\delta)+\frac{m^2\,\delta}{\sin^2\theta'}(l\,U_z-g\,s_z\,\delta)=0 \end{aligned}\right\} \quad . \quad . \quad (19)$$

The solution may be expressed

$$\left.\begin{aligned} & x=\frac{m\,(l_{\theta'}-a\,g\,\gamma\,l\,\delta)\left[a^2\,\alpha^2-i\,\nu\,(a\,\delta\,Q_z/Q-2\cot\theta')\right]}{2\sin\theta'\left[a^4\,\alpha^4+\nu^2\,(a\,\delta\,Q_z/Q-2\cot\theta')^2\right]}\times \\ & \left\{1\pm\left[1-\frac{4\delta\,(l\,U_z-g\,\delta\,s_z)}{(l_{\theta'}-a\,g\,\gamma\,l\,\delta)^2}\,(a^2\,\alpha^2+i\,\nu\,(a\,\delta\,Q_z/Q-2\cot\theta'))\right]^{\frac{1}{2}}\right\} \end{aligned}\right\} . \quad (20)$$

For a Cartesian coordinate system and for $\nu = 0$, Eq. (20) reduces to Eq. (21) of the writer's earlier paper (Fleagle 1955). In common with the result of the earlier paper, Eq. (20) shows that instability is possible for a range of δ between zero and the slope of the isentropic surface. The vertical component of velocity is evidently a quantity crucial to an understanding of development of large-scale disturbances. Further, the simplicity of Eq. (20) is such that one may infer that even rather rough estimates of δ can lead to valid conclusions concerning baroclinic stability and instability.

By the method of particle dynamics it can be shown in a straightforward manner that for δ less than zero or greater than the slope of the isentropic surface potential energy is increased at the expense of kinetic energy (Eady 1949); consequently, under these conditions the disturbance is damped.

The main additional generalization which may be drawn from Eq. (20) is that introduction of a finite latitudinal (north-south) dimension results in the possibility of additional conditions of instability; these effects, however, are comparatively small. Further discussion of instability conditions will be deferred until the slope of the stream surface has been eliminated.

In order to eliminate δ from Eq. (19) or Eq. (20) the perturbation equations may be applied to a spherical coordinate system whose θ, λ surface is horizontal. Then δ may be expressed by the ratio of w to v evaluated at the surface of its maximum value.

The procedure to be followed is to substitute Eq. (12) into Eqs. (6), (8), (9), (10) and (11) including the terms in w. Then if QB is expressed in terms of $(QC)_\tau$ and D_τ, to the same approximation used in deriving Eq. (19) and for disturbances more than several hundred kilometres in length, there results

$$\left.\begin{aligned} & i\,QB\left[-l_\theta/a+\frac{x\sin\theta}{am}\left(\frac{m^2-\cos^2\theta}{\sin^2\theta}+\nu^2\right)\right] \\ & -D_\tau\left[\frac{x^2}{gm}(i\,\nu\sin\theta+\cos\theta)-\frac{l\sigma}{g}\right]-(QC)_\tau\left(i\,l+\frac{\nu\,x\sin\theta}{m}\right)=0 \end{aligned}\right\} \quad . \quad (21)$$

Also, if B is expressed in terms of D_τ and C, there results

$$\left.\begin{aligned} & QB\left[\frac{l\,U_\tau}{g}\left(l-\frac{i\,\nu\,x\sin\theta}{m}\right)+\frac{\gamma\,a\,l^2\,x\sin\theta}{m}\right] \\ & +\frac{i\,x\,D_\tau}{g}\left(l-\frac{i\,\nu\,x\sin\theta}{m}\right)-QC\,s_\tau\left(l-\frac{i\,\nu\,x\sin\theta}{m}\right)=0 \end{aligned}\right\} \quad . \quad . \quad (22)$$

And, if D_τ is expressed in terms of $(QB)_\tau$

$$i\,m\,D_\tau-a\,l\sin\theta\,(QB)_\tau=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

At the stream surface of greatest slope it is easily shown that $(QC)_\tau = \delta\,(QB)_\tau$. This s an independent condition which serves the function of boundary conditions in a hydro-lynamic problem free of internal constraints. The use of this condition permits elimina-ion of D_τ and $(QB)_\tau$ from Eqs. (21), (22) and (23). This gives

$$QB\left[-\frac{l\theta}{a}+\frac{a\alpha^2 x\sin\theta}{m}\right]=(QC)_\tau\left[\frac{a\,l\sin\theta}{g\,m\,\delta}\left(\sigma l-\frac{x^2\cos\theta}{m}\right)\right.$$
$$\left.+\,l-\frac{i\nu x\sin\theta}{m}\left(1+\frac{a\,l\,x\sin\theta}{g\,m\,\delta}\right)\right] \quad . \quad . \quad (24)$$

$$QB\left[\frac{l\,U_\tau}{g}\left(l-\frac{i\nu x\sin\theta}{m}\right)+\frac{\gamma\,a\,l^2 x\sin\theta}{m}\right]$$
$$+\,(QC)_\tau\frac{a\,l\,x\sin\theta}{g\,m\,\delta}\left(l-\frac{i\nu x\sin\theta}{m}\right)=QC\,s_\tau\left(l-\frac{i\nu x\sin\theta}{m}\right) \quad . \quad . \quad (25)$$

From Eqs. (24) and (25), $(QC)_\tau$ may be eliminated, yielding a quadratic equation in δ. Discussion of orders of magnitude shows that one root is nearly zero; this root is of little nterest. The other root may be more easily obtained if it is recognized that at the stream surface of maximum slope the two terms on the left-hand side of Eq. (24) are opposite n sign and of about the same magnitude so that their sum is an order of magnitude smaller han either taken individually. This requires $(QC)_\tau$ to be of the order of magnitude 10^{-10} gm cm^{-3} sec^{-1}. It follows that the second term in Eq. (25) is an order of magnitude maller than the first or third and permits writing

$$\frac{C}{B}=\frac{l\,U_\tau\,[l-i\,(\nu/m)\,x\sin\theta]+g\,\gamma\,a\,l^2 x\sin\theta}{g\,s_\tau\,[l-i\,(\nu/m)\,x\sin\theta]} \quad . \quad . \quad (26)$$

Then if the denominator is rationalized and $(\nu/m)^2 x^2\sin^2\theta$ neglected compared to l^2

$$\frac{C}{B}=\frac{l\,(m\,U_\tau+g\,\gamma\,a\,x\sin\theta)}{g\,m\,s_\tau}+\frac{i\,\nu\,\gamma\,a\,x^2\sin^2\theta}{m^2\,s_\tau} \quad . \quad . \quad (27)$$

Eq. (27) shows that a phase difference between w and v occurs if the disturbance is finite n latitudinal extent ($\nu > 0$) or if the disturbance is unstable (x complex). In either ase the field of vertical velocity is displaced to the west of the field of meridional velocity so that upward motion prevails in the pressure trough, downward motion in the pressure idge. The real part usually is an order of magnitude greater than the imaginary part, so that upward motion usually extends a relatively short distance to the west of the pressure rough, but this may not always be so.

For w and v in phase, the ratio C/B represents the slope of the stream surface. When nstability is large so that there exists an appreciable phase angle between w and v, δ ecomes a function of λ. Throughout most of the wavelength δ usually is positive, but n the regions just west of the pressure trough and ridge δ may be negative. It follows rom the discussion of Eq. (20) that in these limited regions, potential energy is increased t the expense of kinetic energy. This process, however, usually is secondary to the nstability of the major part of the disturbance so that in the interest of simplicity, it may e neglected so long as the results of the simpler analysis are not extrapolated to conditions f large phase difference between w and v. The following development, therefore, is ccurate for small instability and great latitudinal extent, less accurate for large instability r small latitudinal extent.

Substitution of the real part of Eq. (20) into the real part of Eq. (27) gives

$$\delta = \frac{2\,l\,U_z\,\alpha^2 + l_\theta\,l\,g\,\gamma/a}{2\,g\,s_z\,\alpha^2 + g^2\,\gamma^2\,l^2} \quad . \quad . \quad . \quad (28)$$

No distinction has been made between U_r and U_z, s_r and s_z, θ' and θ. Substitution of Eq. (28) in Eq. (20) neglecting the small terms in $i\,\nu$ in Eq. (20) gives for the real and imaginary parts, respectively

$$\sigma_R - m\,K = -\frac{m\,(l^2\,\gamma\,U_z - l_\theta\,s_z/a)}{a\sin\theta\,(2\,s_z\,\alpha^2 + g\,\gamma^2\,l^2)} \quad . \quad . \quad . \quad (29)$$

$$\sigma_i = \frac{m\,l\,\gamma\,(l\,U_z - s_z\,l_\theta/a\,l\,\gamma)^{\frac{1}{2}}}{a\sin\theta\,(2\,s_z\,\alpha^2 + g\,\gamma^2\,l^2)}\left\{l\,U_z + \frac{l_\theta}{a}\left(\frac{s_z}{l\,\gamma} + \frac{g\,\gamma\,l}{\alpha^2}\right)\right\}^{\frac{1}{2}} \quad . \quad . \quad (30)$$

The properties of Eqs. (29) and (30) are similar to those discussed in the earlier paper, but the differences introduce results of importance for the discussion of the general circulation.

5. Characteristics of growing disturbances

The instability criterion is obtained from the radical in Eq. (30). Instability exists for

$$l\,U_z > -\frac{l_\theta}{a}\left(\frac{s_z}{l\,\gamma} + \frac{g\,\gamma\,l}{\alpha^2}\right) \quad . \quad . \quad . \quad (31)$$

This inequality shows that at a particular latitude there is a minimum vertical shear required for instability; long waves are stable, whereas short waves may be unstable and the effect of hydrostatic stability is toward baroclinic stability. These results are in agreement with the results of the writer's earlier study of baroclinic instability, and they are in general agreement with the results of Charney (1947), Kuo (1952) and others. In addition, Eq. (31) shows that the effect of finite as opposed to infinite latitudinal extent of

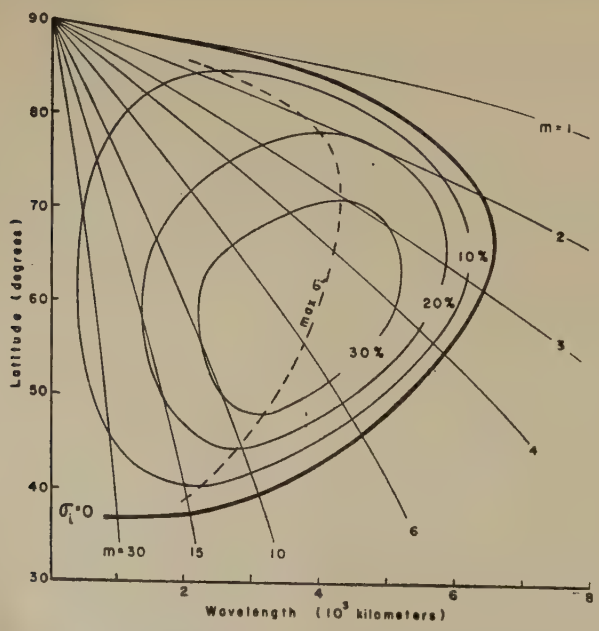

Figure 2. Amplification in 24 hr (labelled in %) computed from Eq. (30) as a function of wavelength (or zonal number) and latitude for the following values of the parameters : latitudinal temperature gradient, $2\sin\phi\cos\phi$ °C (deg lat.)$^{-1}$; temperature, 260°K; lapse rate, 6·5°K km^{-1}; $K = 0{\cdot}6 \times 10^{-6}$ sec^{-1}.

disturbances may be toward instability, and that for a particular vertical shear there is a critical latitude below which baroclinic disturbances are stable.

In order to discuss the rate of growth of baroclinic disturbances on a spherical earth, it is convenient to plot σ_i from Eq. (30) as a function of various combinations of parameters. Adiabatic change of state is assumed, except as noted. In Fig. 2, σ_i is shown as a function of latitude for an undisturbed latitudinal temperature gradient which varies as the product of the sine and cosine of the latitude. An arbitrary value of 1°C per degree of latitude at 45° lat has been chosen. This distribution is in rough conformity with the mean temperature distribution in the middle troposphere in winter. Instability increases with latitude from the critical latitude of 37° to about 60° and then decreases to zero at the pole. The main qualitative features are not critically dependent on the choice of temperature distribution although another choice may change considerably the rate of amplification and may displace the line of most rapid growth somewhat. A latitudinal wave number, ν, of four has been used. The longitudinal wave number of maximum instability decreases with latitude to about 80° and is constant from there to the pole. It is to be expected that the atmosphere selects the wave number of maximum instability unless this is prevented by external constraints or by the requirement of continuity within the atmosphere. Therefore, the number of disturbances should decrease with latitude from about ten at 45° to three at 70° and two at 80°, but variation about these figures must be expected. The intensity of the disturbances should be greatest at about 60°. These conclusions are in rough agreement with observations.

In spite of the fact that equation Eq. (30) is based entirely on linear theory, Fig. 2 makes clear why intense disturbances develop into vortices rather than into more and more intense pressure troughs. Because the most highly favoured wave number decreases with latitude to a minimum of two, the number of growing disturbances of great latitudinal extent should not be expected to exceed two or three at any one time. These disturbances can develop only between their critical latitude and the pole; and they develop most rapidly at about 60° lat., so that those which grow sufficiently will exhibit minimum pressure at 60° and the velocity field will be vortical. Other disturbances are severely limited in latitudinal extent, so that they too assume the form of vortices if their instability is sufficient for them to reach that state.

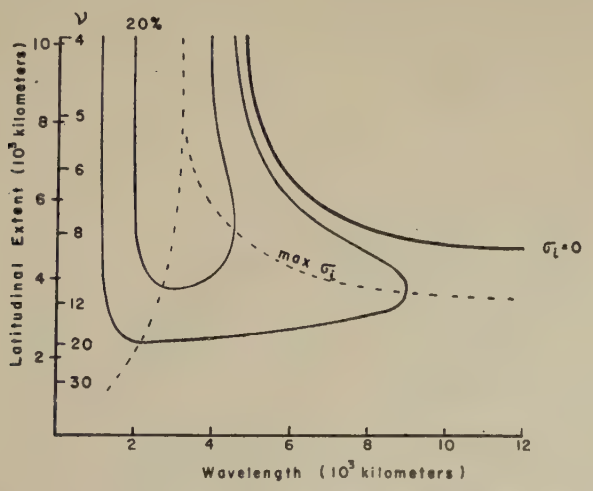

Figure 3. Amplification in 24 hr (labelled in %) computed from Eq. (30) as a function of wavelength (or zonal wave number) and latitudinal extent (or meridional wave number) for the following values of the parameters : $\phi = 45°$ lat.; $U_z = 2{\cdot}5 \times 10^{-3}$ sec^{-1}; $K = 0{\cdot}6 \times 10^{-5}$ sec^{-1}; lapse rate, 6·5°K km^{-1}.

The dependence of the rate of development on latitudinal extent is shown in Fig. 3 for 45° lat., vertical shear of $2{\cdot}5 \times 10^{-3}$ sec^{-1}, and a temperature lapse-rate of 6·5°K km^{-1}. Fig. 3 shows that disturbances of greatest possible latitudinal extent ($\nu = 4$) grow most rapidly; taken together with the inferences drawn from Fig. 2, this indicates that the atmosphere favours the development of a small number of large-amplitude disturbances of maximum latitudinal extent which are centered at about lat. 60°. Vortices with smaller latitudinal extent should be expected to form between latitudes 30° and 60°.

An additional important inference may be drawn from Fig. 3. For disturbances whose wavelength is prescribed, perhaps by the distribution of land and ocean or by the requirement of an integral number of disturbances at a particular latitude, maximum rate of growth may occur for a limited latitudinal extent. For these conditions, vortices should be expected to develop in middle latitudes with higher pressure toward the pole. The phenomena known as ' cut-off ' lows and ' blocking ' highs may be evidences of this type of development.

The dependence of the rate of development on hydrostatic stability is shown in Fig. 4 for 45° lat., vertical shear of $2{\cdot}5 \times 10^{-3}$ sec^{-1}, and latitudinal wave number of four. The rate of development increases very strongly with decrease in hydrostatic stability, and there is a marked preference for disturbances of intermediate length. Therefore, development is much more rapid in saturated air than in unsaturated air. This may be an important reason for the observation that greater development occurs in areas of low pressure (where upward motion predominates in growing disturbances) than in areas of high pressure.

Figs. 2, 3 and 4 utilize arbitrary choices of the parameters which appear in Eq. (30). Other choices, of course, would alter the numerical results, but the main conclusions are fairly stubborn, and can be reversed only by making very extreme choices of the parameters. However, the behaviour of real disturbances is far from simple. The true complexity of the problem of describing a particular baroclinic development may be apprehended, at least vaguely, if one imagines a superposition of linear disturbances in θ and λ which together constitute a real disturbance at some initial time. The subsequent behaviour depends on the sum of the corresponding calculations from equations of the form of Eqs. (29) and (30).

Finally, a reservation should be stressed. In eliminating the necessity for external boundary conditions by applying the equations at the stream surface of greatest slope,

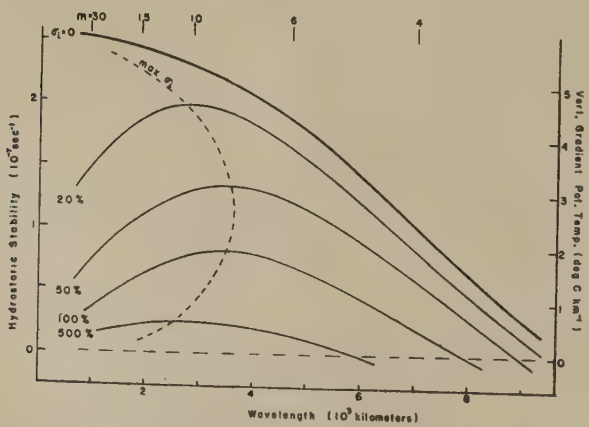

Figure 4. Amplification in 24 hr (labelled in %) computed from Eq. (30) as a function of wavelength (or zonal wave number) and hydrostatic stability (or vertical gradient of potential temperature) for the following values of the parameters : $\phi = 45°$ lat.; $U_z = 2{\cdot}5 \times 10^{-3}$ sec^{-1}; $K = 0{\cdot}6 \times 10^{5}$ sec^{-1}; $\nu = 4$.

no lateral boundary conditions have been needed; stated differently, the influence of conditions at one latitude on the development at other latitudes has been ignored. The reason that this has been done is that the procedure leads to simple answers, whereas a more general solution appears to be very much more difficult; the procedure must be justified by testing the agreement of theory and observation.

6. The development of jet streams

The results of the previous section provide considerable insight into the relation between baroclinic instability and the development and maintenance of 'jet streams' – the high-speed, relatively narrow, currents which are characteristic of many well-developed baroclinic disturbances. Consider the following hypothetical case of jet-stream development in the northern hemisphere. Initially, flow is nearly zonal with small-amplitude disturbances, latitudinal profile of wind and temperature is smooth and may be unspecified, except that the temperature gradient is sufficient, that north of a critical latitude, baroclinic instability exists as expressed by Eq. (30). As development proceeds, marked meridional flow is established north of the critical latitude, whereas the flow south of the critical latitude remains essentially zonal. The flow pattern exhibits confluence and diffluence in the region north of the critical latitude. Eq. (28) shows that for very short disturbances the slope of the stream surface equals that of the isentropic surface; that is, the air moves along the isentropic surface and the latter does not move. For increasing length of disturbances the slope of the stream surface becomes increasingly less than the slope of the isentropic surface. For the wavelength of greatest instability, Eq. (28) shows that the air moves at about half the slope of the isentropic surface. This requires that isotherms move horizontally at about half the speed of the horizontal wind. It follows that as meridional currents develop in the regions of pressure troughs, isotherms on horizontal or constant pressure charts are crowded southward against the critical latitude. The thermal-wind relationship (Eq. 23 and hydrostatic equation) expresses the accompanying development of the jet stream in the upper troposphere above this region of increasing baroclinicity. So the jet stream is a direct and rather simple consequence of baroclinic instability and its distribution with latitude.

The increased baroclinicity conceivably may lead to southward extension of the region of baroclinic instability. However, the latitudinal cut-off is fairly sharp, so that penetration is likely to be limited to several degrees of latitude. The critical latitude (ϕ_c) may be determined by making inequality Eq. (31) an equality and by solving for ϕ_c. The approximate result is the transcendental equation

$$\sec \phi_c - \cos \phi_c = \frac{s_z}{2(\omega + K) a \gamma U_z} \quad . \quad . \quad . \quad (32)$$

For lapse rate of 6·5°K km^{-1} and adiabatic change of state, an increase of vertical shear from 2·5 to 5 m sec^{-1} km^{-1} results in southward displacement of the critical latitude from about 38° to 29°. Similar southward displacement would result from halving the hydrostatic stability (e.g., increasing lapse rate from 6·5 to 8·2°K km^{-1}) holding the vertical shear constant. These calculations indicate that the region between 30°S and 30°N is strongly protected from the penetration of baroclinic disturbances, except under conditions of very weak hydrostatic stability.

In northward meridional currents isotherms move northward, and if the disturbance is limited in latitudinal extent, increasing temperature gradient may occur in the region of confluence between the pressure ridge to the south and the trough to the north. A jet stream therefore may develop at high latitude.

Eqs. (29) and (30), amplified by the preceding discussion, should enable a competent physicist to sketch an idealized upper-air chart without having ever seen one or heard one described. Given a description of the undisturbed model he should be able to determine the critical latitude, latitudinal and longitudinal dimensions of disturbances, locations of greatest intensity, locations of jet streams, etc. In Fig. 5 an idealized chart is illustrated; it was not, however, sketched by the naive physicist. At the critical latitude a zone of great baroclinicity has developed in the base of each pressure trough, and this zone marks the approximate location of the jet stream. The long-wave disturbance on the right of Fig. 5 is limited in latitudinal extent in conformity with Fig. 3, and a jet stream has developed between the ridge on the south and the pressure trough on the north.

Eqs. (31) or (32) make possible certain calculations for planetary atmospheres other than the earth. For example, the following parameters for the Martian atmosphere may be extracted from data given by Hess and Panofsky (1951): $2\omega = 1{\cdot}42 \times 10^{-4}$ sec^{-1}, $g = 370$ cm sec^{-2}, $a = 3370$ km, ratio of specific heats $= 1{\cdot}4$, specific heat at constant pressure $= 10^7$ erg g^{-1} deg^{-1}, $\phi_c = 40°$ (winter) and 50° (summer), $U_z = 4{\cdot}0 \times 10^{-3}$ sec^{-1} (winter) and $1{\cdot}6 \times 10^{-3}$ sec^{-1} (summer). Substitution of these numerical values in Eq. (32) gives for the Martian middle-latitude tropospheric lapse rate: 1°C/km in winter and 2°C/km in summer.

7. The region between the critical latitudes

The preceding discussion indicates that in a very important sense the behaviour of the region between the critical latitudes of northern and southern hemispheres is

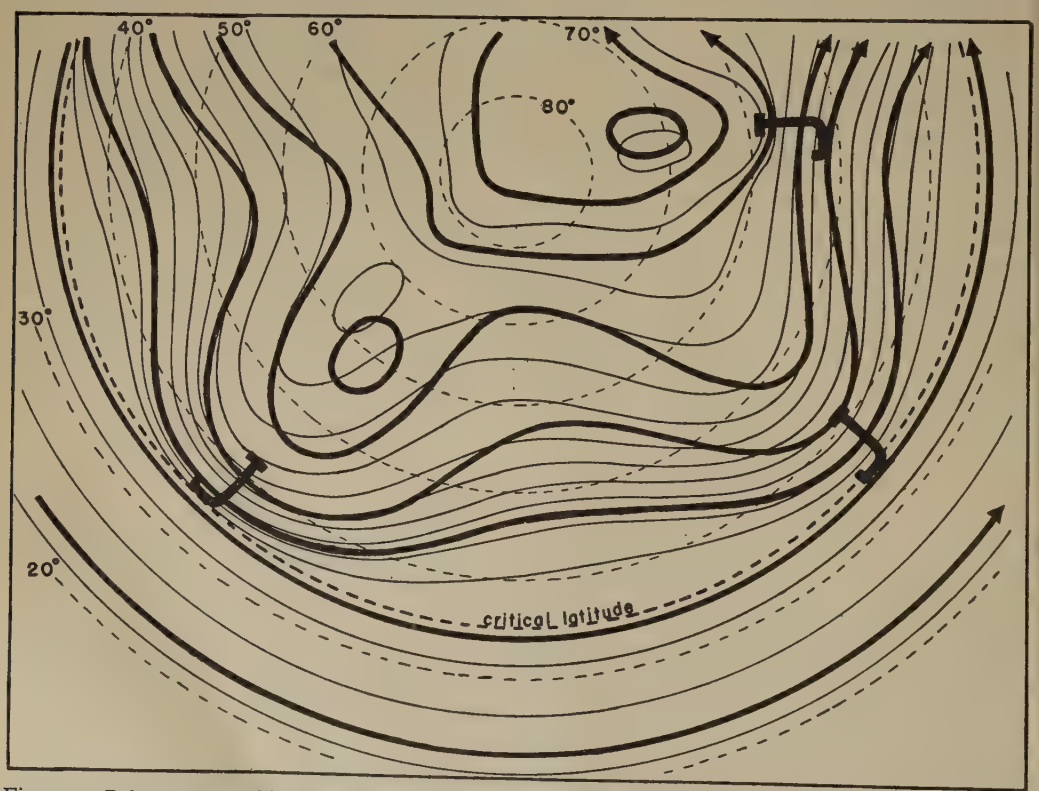

Figure 5. Polar stereographic projection showing the idealized development of jet streams (J). Heavy lines represent streamlines, thin lines represent isotherms. On the left, a disturbance of greatest possible latitudinal extent exhibits a jet stream at the critical latitude; on the right, a longer disturbance of limited latitudinal extent exhibits two jet streams.

independent of the regions on the poleward side of the critical latitudes. The mechanism for development of a strong zonal current is absent here, so one may imagine the tropical atmosphere as being ' frictionally driven ' by the jet stream. The horizontal shear which would develop would be limited only by ' frictional ' stresses or by the onset of some other form of instability associated with other properties of the general circulation not considered so far, perhaps the horizontal shear itself. In this section the stability of disturbances in this region is considered.

The analysis of the middle-and high-latitude region which has been completed may be extended to the low-latitude region if K, the parameter which determines the undisturbed zonal speed, is allowed to vary with latitude. For a region of anticyclonic shear, K is negative, and l decreases with increasing anticyclonic shear. Eq. (28) shows that as the anticyclonic shear approaches the Coriolis parameter, or the absolute vorticity of the undisturbed wind approaches zero, the slope of the stream surface approaches the slope of the isentropic surface. Eq. (20) shows that this is a condition of baroclinic stability, and Eq. (30) shows this stability explicitly. Thus the effect of anticyclonic shear is toward baroclinic stability, whereas the effect of cyclonic shear is towards instability. Therefore, as the jet stream develops just poleward of the critical latitude, the developing latitudinal shear accentuates the sharp distinction between stable and unstable regions. A virtual wall of stability is created in this way which offers great resistance to penetration of the jet stream to lower latitudes.

The conclusions just stated must be resolved with the contrary conclusions which spring from the criterion for inertial (sometimes called centrifugal or dynamic) instability. According to this criterion, flow in which anticyclonic shear exceeds Coriolis parameter is unstable for small disturbances. Efforts have been made in the past to account for the development of cyclones and anticyclones (now attributed to baroclinic instability) on this basis, and it appears plausible that inertial and baroclinic instability might interact in a positive sense, so that the combined instability would be increased by anticyclonic shear. That this is not so must be recognized from Eq. (30). Evidently, the criterion of inertial instability is inapplicable to large-scale disturbances in hydrostatically stable air.

However, there is no reason to discard the criterion as applied to smaller-scale disturbances. Baroclinic instability is negligible for very short wavelengths, so that the field is left open for inertial instability. These considerations suggest that the anticyclonic shear associated with the jet stream may be limited to the value of the Coriolis parameter

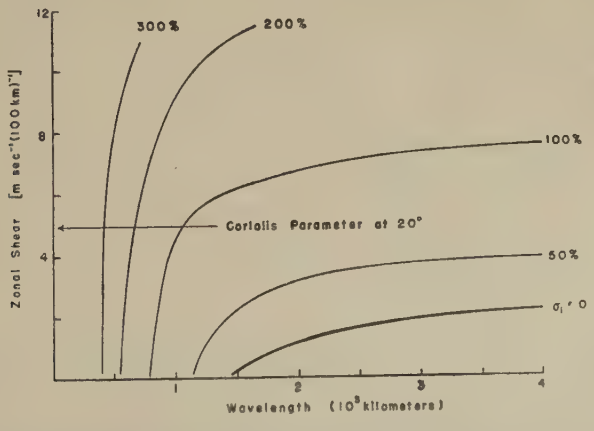

Figure 6. Amplification in 24 hr (labelled in per cent) computed from Eq. (33) as a function of wavelength and zonal shear for the following values of the parameters : $\phi = 20°$ lat., $\nu = 0$, $U_z = 10^{-4}$ sec^{-1}.

by small-scale instability. One might infer, therefore, that the undisturbed velocity profile below the critical latitude should exhibit anticyclonic shear not greater than the Coriolis parameter and that the shear should decrease monotonically with distance from the critical latitude. Observations appear to be consistent with these suppositions.

In localized areas where the hydrostatic stability is negligible unstable quasi-horizontal disturbances may form even at low latitudes. In such areas Eq. (30) may be written

$$\sigma_i = \frac{m\left[U_z\left(U_z + g\,\gamma\, l_\theta/a\,\alpha^2\right)\right]^{\frac{1}{2}}}{a\,g\,\gamma\cos\phi} \quad . \quad . \quad . \quad . \quad . \quad (33)$$

The amplification expressed by Eq. (33) is illustrated in Fig. 6.
If $\sin^2\phi$ is neglected compared to m^2, instability exists for

$$U_z\left(\frac{m^2}{\cos^2\phi} + \nu^2\right) > 2\,(\omega + K)\,a\,g\,\gamma\cos\phi \; . \quad . \quad . \quad (34)$$

In the region of anticyclonic shear, $(\omega + K)\cos\phi$, may be very small in spite of the low latitude; in this case the instability is directly related to the baroclinicity, and essentially all waves are unstable. However, it is probably more realistic to consider that the necessary condition of negligible hydrostatic stability occurs only over small areas where the air is saturated; here development of short waves can occur. Instability is favoured by baroclinicity and by anticyclonic shear : because these parameters normally increase with latitude and because even a very small hydrostatic stability exerts a strong stabilizing effect near the equator, easterly waves and hurricanes are more likely to develop at 10 or 20 degrees latitude than near the equator. In summary, the factors which favour growth of quasi-horizontal disturbances in the region below the critical latitude are : (1) negligible hydrostatic stability, (2) baroclinicity and (3) anticyclonic shear.

Because the criterion for development of tropical disturbances is simple and explicit, it may be feasible to substitute observations in Eq. (30) in order to plot charts indicating areas of incipient or possible development.

8. The wave speed

Eq. (29) shows that the speed of unstable disturbances relative to the undisturbed wind speed increases with baroclinicity and with wavelength. The dependence on latitude is rather complicated. Because the height of the stream surface varies with wave number and latitude as shown later, the wave speed relative to the earth's surface is still more complicated and can be described only after the height of the stream surface has been determined. Complete discussion of this problem requires a complicated integration of the confluent hypergeometric equation, but a simple discussion based partly on results of observation is sufficient for heuristic purposes. Eq. (24) may be integrated from the ground surface to the stream surface of maximum slope, and small terms including the phase shift between $(QC)_r$ and B may be neglected, giving

$$\frac{(QC)_{(h)} - (QC)_{(0)}}{(QB)_{(h)}} = \frac{h\,\overline{QB}}{2\,a\,l\,(QB)_{(h)}}\left\{\frac{a^2\,\alpha^2\sin\theta}{m}\left(2\,x + \frac{m\,h\,U_z}{a\sin\theta}\right) - 2\,l_\theta\right\} \quad . \quad (35)$$

The angular frequency (x) may be eliminated by substitution of the real part of Eq. (20). This yields a cubic equation in δ. In principle, the solution to this equation may be equated with Eq. (28) in order to express h explicitly. However, for the purpose of this discussion it is sufficient to consider only the special case of ν small compared to $m/\sin\theta$;

hat is, the case of latitudinal extent large compared to wavelength. For this case, combination of the real parts of Eqs. (20) and (35) gives

$$\delta = \frac{h\,(a\,\alpha^2\,h\,U_z - l_\theta)}{a\,l\,(g\,\gamma\,h + 2\,(QB)_{(h)}/\overline{QB})} \quad . \quad . \quad . \quad (36)$$

On the basis of observations the ratio $(QB)_{(h)}/\overline{QB}$ may be estimated to be less than unity. f the arbitrary value of one half is chosen, Eq. (36) may be equated with Eq. (28), giving quadratic equation in h. The roots are approximately

$$h = \pm\frac{l}{\alpha}\left[\frac{2\,\alpha^2\,U_z + g\,\gamma\,l_\theta/a}{g\,U_z\,(2\,s_z\,\alpha^2 + g\,\gamma^2\,l^2)}\right]^{\frac{1}{2}} \quad . \quad . \quad . \quad (37)$$

Only the positive root is to be used. Substitution in Eq. (29) gives for the angular velocity f the disturbance

$$\sigma_R - \frac{m\,U_{(0)}}{a\sin\theta} = \frac{m\,l}{a\,\alpha\sin\theta}\left[\frac{U_z\,(2\,\alpha^2\,U_z + g\,\gamma\,l_\theta/a)}{g\,(2\,s_z\,\alpha^2 + g\,\gamma^2\,l^2)}\right]^{\frac{1}{2}} - \frac{m\,(l^2\,\gamma\,U_z - l_\theta\,s_z/a)}{a\sin\theta\,(2\,s_z\,\alpha^2 + g\,\gamma^2\,l^2)} \quad . \quad (38)$$

nd for the wave speed

$$c_R - U_{(0)} = \frac{l}{\alpha}\left[\frac{U_z\,(2\,\alpha^2\,U_z + g\,\gamma\,l_\theta/a}{g\,(2\,s_z\,\alpha^2 + g\,\gamma^2\,l^2)}\right]^{\frac{1}{2}} - \frac{l^2\,\gamma\,U_z - l_\theta\,s_z/a}{2\,s_z\,\alpha^2 + g\,\gamma^2\,l^2} \quad . \quad . \quad (39)$$

The wave speed computed from Eq. (39) is illustrated in Fig. 7. At 45° lat., wave speed elative to sea-level undisturbed flow is positive, and exhibits a maximum at about the wavelength of maximum instability. The wave speed corresponds closely to that found by Kuo (1952), except for wavelengths greater than about 3,500 km. At 60° lat., the wave peed is greater than at 45°, and unstable disturbances of greater length are possible. Several implications of interest follow from Fig. 7.

First, the group velocity for superimposed disturbances of more than one wavelength s given by

$$G = c_R - L\frac{\partial c_R}{\partial L} \quad . \quad . \quad . \quad . \quad (40)$$

The group velocity computed from Eqs. (39) and (40) for the 45° latitude curve is shown n Fig. 7. The group and phase velocities are equal at about 3,100 km; it follows that

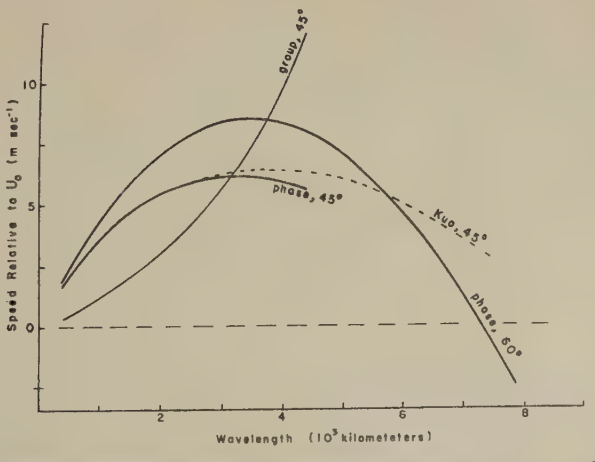

Figure 7. Phase and group velocities relative to sea-level undisturbed flow as a function of wavelength computed from Eq. (39) for latitudes 45° and 60° and for $U_z = 2{\cdot}5 \times 10^{-3}$ sec^{-1}; $K = 0{\cdot}6 \times 10^{-5}$ sec^{-1}; apse rate, 6·5°K km^{-1}. The phase velocity computed by Kuo (1952) for 45° is shown by the dashed line.

dispersion is small for the waves of most rapid growth. Dispersion for very long or very short waves is considerable, so that these waves must change rapidly in shape and amplitude independently of the release of potential energy. However, disturbances of maximum growth rate should be identifiable for extended periods of time, for the most part they should move with the phase speed calculated from Eq. (39), and their early growth should conform to calculation based on Eq. (30).

Second, the increase of wave speed with latitude shows that a pressure trough or ridge in middle latitudes should gradually 'tilt' from a meridional orientation toward a NE-SW orientation. On the other hand, at the pole the wave speed vanishes; consequently, there is a latitude of maximum wave speed. The accuracy of Eq. (39) probably is insufficient to justify a determination of the latitude of maximum wave speed. However, the qualitative result provides an explicit explanation for the observation (Widger 1949 and others) that over most of the northern hemisphere the meridional and zonal velocity components are correlated positively but that in the polar region they are correlated negatively. These correlations provide the concept of northward transport of angular momentum in the region of positive correlation and southward transport in the region of negative correlation.

9. Conclusions

Here an effort will be made to trace in qualitative fashion the thread of cause and effect from the primary cause of the general circulation, differential heating by solar radiation, to the dominant features of the general circulation. It is a consequence of the spherical shape of the earth, and absorption by its atmosphere, that the average solar-radiation intensity at the earth's surface is roughly proportional to the square of the cosine of the latitude. If one assumes surface temperature to be roughly proportional to insolation, it follows that the latitudinal temperature gradient at the earth's surface should be roughly proportional to the product of the sine and cosine of the latitude : that is, greatest in middle latitudes and least in polar and equatorial regions. The solar energy absorbed at the earth's surface is efficiently transferred upward by the processes of radiation, turbulent conduction, and the evaporation-condensation cycle, all processes outside the scope of this investigation.

This study has shown that the temperature distribution just described is unstable for disturbances poleward of a critical latitude and shorter than a critical length. Growth is most rapid for disturbances of intermediate length – three to five thousand kilometres in middle latitudes. The growth of disturbances may be understood as a consequence of the conversion of potential energy of mass distribution to kinetic energy through sinking of the cold air as it moves equatorward and rising of the warm air as it moves poleward. The fact that maximum energy can be converted for baroclinic disturbances of lengths of three to five thousand kilometres explains the preference of the atmosphere for disturbances of this size. It also follows that the number of growing disturbances must decrease toward the pole from about ten at 45° lat. to three at 70°.

If disturbances at all latitudes are considered, those at about 60° lat. should develop most rapidly, attain greatest intensity and therefore convert potential to kinetic energy most rapidly. The amplitude of baroclinic disturbances must vanish at the poles and at the critical latitudes; therefore, marked development in middle latitudes must result in formation of closed vortices. There is a preference, on the basis of growth rate, for disturbances which extend from the pole to the critical latitude; however, this preference can be satisfied only by two disturbances, so that all others should be smaller in latitudinal extent.

Instability is accompanied by horizontal temperature advection at roughly half the speed of the wind; it follows that in the region just poleward of the critical latitude,

sotherms are crowded together in the base of pressure troughs. The thermal-wind quation expresses the accompanying development of a high-speed zonal current (jet tream) in the upper troposphere. The stabilizing effect of latitude presents a rather sharp ut-off to development below the critical latitude, and this is augmented by the stabilizing ffect of anticyclonic shear.

Below the critical latitude unstable disturbances may develop in areas where the ydrostatic stability nearly vanishes. Here, rapid growth is favoured by baroclinicity nd anticyclonic shear of the undisturbed wind. Areas of saturation in the trade wind egion appear to be the most favourable for instability of this sort.

Dispersion of the most unstable baroclinic disturbances is very small, so that change f form by dispersion appears to be a secondary, though not negligible, effect. It may be nferred from this that the lifetime as well as the size of individual disturbances is letermined mostly by the available potential energy, rather than by a chaotic and more r less random superposition of disturbances drawn from a very large population.

The results summarized above lead to the conclusion that the major features of the arge-scale circulation are understandable in terms of explicit quantitative parameters. The impact of this conclusion is to emphasize that there is little reason to regard the time r space average of the large-scale circulation as being more fundamental or more easily understood than the details of the circulation – indeed the reverse appears to be true. To consider statistical properties of the general circulation to the exclusion of specific properties is to blur unnecessarily the outlines of the problem and for this reason, on a, deeper intuitive level, the statistical approach to understanding of the general circulation is unsatisfying. Cyclone development and the associated features of the general circulation are specific effects; one feels impelled to search for equally specific causes. There are, of course, important uses to which statistical calculations of momentum, energy, or mass transfer may be put.

In this paper the attempt has been made to build bridges at certain points between observation and the theory of planetary aerodynamics, so that access from one to the other may result in advances in understanding. Some success may be claimed in this endeavour, but there remains much that is not understood, and more that is understandable only if the problem is placed on the platform of an arbitrarily defined undisturbed atmosphere. A proper conclusion to this study of the general circulation must direct attention to those aspects of the problem which have been omitted from consideration or which have been incompletely treated.

First, the physical processes of radiation, phase change, molecular and turbulent conduction, and frictional stresses have not been considered.

Second, the study has been limited to a single stream surface located in the troposphere; vertical structure has been introduced only by inference. The pronounced baroclinic zone at the critical latitude requires a jet stream in the upper troposphere only if the sea-level zonal-wind distribution is specified. A mathematical result which would include the sea-level distribution of wind would require consideration of surface-frictional stresses. In the absence of a theory which includes friction it may be imagined that friction acts to make the undisturbed sea-level velocity nearly zero, this conclusion following from the concept of least action applied to the energy expended through friction.

The normal distribution of temperature and wind velocity in the stratosphere is an important aspect of the general circulation which is not included in the results of this study. The main features of this distribution may be products of the general circulation, or they may result mostly from radiation. Thorough understanding of the general circulation scarcely can be claimed until this problem has been answered.

Finally, many details of the general circulation, which may be very important in spite of being called *details*, are not included in the results. Pressure-jumps, frontal-surfaces, the tropopause and other discontinuities are essentially non-linear properties, so that they cannot appear in a linear treatment of the hydrodynamic equations. And details of vertical structure such as change in intensity with height, slope of baroclinic zones and pressure systems, and the fine structure of jet streams have not been considered.

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On the heat balance and maintenance of circulation in the trades

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SUMMARY

The heat balance is calculated for the Pacific north-east trade during summer and it is shown that the trade exports sensible as well as latent heat. The sensible-heat gain is related to the surface pressure distribution; it is found that the low-level circulation in the region considered is self-driving.

1. INTRODUCTION

Several years ago (Riehl, Yeh, Malkus and LaSeur 1951) the authors and collaborators attempted (a) to describe the structure of the north-east trade in the Pacific Ocean between the surface and 3 km, (b) to calculate the heat, moisture and momentum balance in this region, and (c) to find a mechanism for the breakdown of the inversion. This paper will extend the former heat-balance computations and discuss the maintenance of the trade winds.

The following symbols are used in the paper :

c_p = specific heat of air at constant pressure
c_v = specific heat of air at constant volume
L = latent heat of condensation
p = pressure
P = precipitation
Q = substantial change in heat content
Q_s = sensible-heat exchange between ocean and atmosphere
R = net loss of heat by radiation
R^* = gas constant for air
s = horizontal space coordinate along air trajectory
t = time
T = temperature
v = component of mean motion along s
w = vertical component of velocity
z = vertical space coordinate
θ = potential temperature
$\kappa = R^*/c_p$
ρ = density
τ = shearing stress in s, z plane
Φ = geopotential

2. HEAT BALANCE

In the previous paper it had been assumed that sensible-heat sources and sinks cancel, an assumption made because of difficulties in calculating the sensible-heat export from the region considered. The sink is the net radiative cooling, the sources are the flow of sensible heat from ocean to atmosphere and the condensation heating. Integrating over

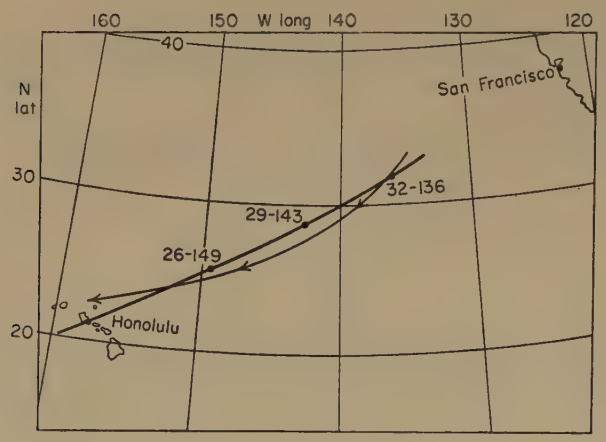

Figure 1. Location of weather ships July-October 1945, and mean air trajectory for period (I).

the mean trajectory from 32°N, 136°W to Hawaii (Fig. 1), and assuming no net heat gain

$$\int\int (Q_s + LP - R)\, ds\, dz = 0 \qquad . \quad . \quad . \quad (1)$$

for balance. Eq. (1) really represents a volume integral. However, the turning of wind with height through the layer was very small, so that, with some approximation (cf. Riehl *et al.* 1951, referred to subsequently as I), the mean motion may be regarded as two-dimensional and unit distance considered along the axis normal to the trajectory in all calculations.

The data satisfied Eq. (1) fairly well. If completely true, the trades act only as accumulators and exporters of latent heat. However, as evident from a cross-section of potential temperature (Fig. 2), Eq. (1) cannot be entirely correct because the mean trajectories cross from low to high potential temperatures, especially below the trade inversion. This indicates some export of sensible heat. Our previous attempt to compute this export failed; a slightly different approach will now be taken by calculating an energy budget by integration, over the section, of the equation expressing the first law of thermodynamics. Per unit mass this law may be stated in the form :

$$Q = c_p \frac{T}{\theta}\frac{d\theta}{dt} = c_p \frac{dT}{dt} - \frac{1}{\rho}\frac{dp}{dt} \qquad . \quad . \quad . \quad (2)$$

This equation can be integrated directly using the data given in (I) because trajectories have been computed. However, a few well-known transformations will be made first in order to facilitate the later analysis. In the steady state considered here

$$\frac{1}{\rho}\frac{dp}{dt} = \frac{v}{\rho}\frac{\partial p}{\partial s} + \frac{w}{\rho}\frac{\partial p}{\partial z}. \qquad . \quad . \quad . \quad . \quad (3)$$

Because of the hydrostatic relation,

$$\frac{w}{\rho}\frac{\partial p}{\partial z} = -\frac{d\Phi}{dt}.$$

The first equation of motion is given by

$$\frac{dv}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial s} + \frac{1}{\rho}\frac{\partial \tau}{\partial z}. \qquad . \quad . \quad . \quad . \quad (4)$$

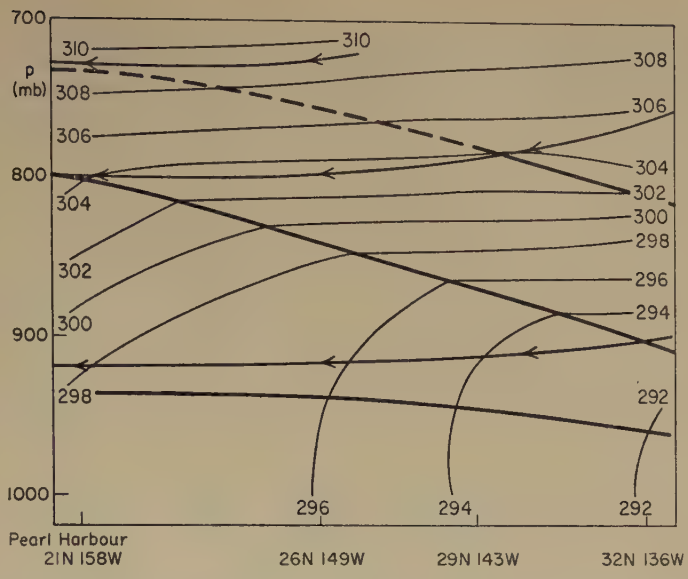

Figure 2. Vertical cross-section of potential temperature (°A) in relation to sample air trajectories (I). The heavy lines denote the top of the subcloud layer, and base and top of the trade-wind inversion.

Using this equation

$$\frac{v}{\rho}\frac{\partial p}{\partial s} = -\left(\tfrac{1}{2}\frac{dv^2}{dt} - \frac{v}{\rho}\frac{\partial \tau}{\partial z}\right),$$

where $\frac{1}{2}\frac{dv^2}{dt} = \frac{dKE}{dt}$, the change of kinetic energy per unit mass. The first law now transforms to

$$\rho\, Q = \rho \frac{d}{dt}(\Phi + c_p\, T + KE) - v\frac{\partial \tau}{\partial z}. \qquad . \qquad . \qquad (5)$$

Table 1 gives the value of all terms, integrated over the section. The first term on the right-hand side of Eq. (5) was computed following trajectories such as shown in Fig. 2 and also by breaking the substantial derivative down into partial space derivatives.

TABLE 1. ENERGY TERMS INTEGRATED OVER THE SECTION (unit 10^9 cal/day)

	+	−
Release of potential energy		7·0
Increase of enthalpy	15·2	
Increase of kinetic energy	0·004	
Work against friction	0·50	
	15·7	7·0
Gain of sensible heat	8·7	

Since a sensible-heat increase is calculated, we must amend Eq. (1) to read :

$$H = \iint (Q_s + LP - R)\, ds\, dz, \qquad . \qquad . \qquad . \qquad (6)$$

where H is the change in heat content integrated over the section.

Since the weather ships did not make rainfall measurements, Eq. (6) cannot furnish a complete check on all calculations but must be solved for the precipitation warming. Table 2 shows the calculation, using values for radiative cooling and flow of heat from ocean to air obtained previously.

TABLE 2. Computation of precipitation heating over the section (unit 10^9 cal/day)

		+	−
Increase of sensible heat		9	
Flow of sensible heat from ocean			3
Net radiative cooling		21	
		30	3
Precipitation heating	27	27	

Computation of the moisture balance will indicate to what extent the result of Table 2 can be considered valid. The moisture balance is presented in Table 3, again using values from the previous paper.

TABLE 3. Computation of moisture balance over the section (expressed in latent heat; unit 10^9 cal/day)

		+	−
Evaporation		49	
Lateral export by mean motion			26
Vertical export through turbulence			5
		49	31
Precipitation heating (from Table 2)	27	18	

Agreement has been attained within the limits of accuracy of the computations. It should be pointed out that the vertical export through turbulence has not been measured. It is a transport assumed in the first paper for moisture balance. If we exclude this amount, the difference of evaporation minus lateral export is 23 units, closer to the value demanded for heat balance in Table 2. Since all calculated values are subject to sizable error, however, one must not expect precise balances and the result of Table 3 may be considered quite satisfactory. Certainly it does not follow that there is no vertical export of moisture through the 3-km level.

Because of the steady state the sensible heat acquired must be exported. The export of sensible heat is 8 units, that of latent heat 26-31 units, adding to a total of 34-39 units. Thus the sensible heat export contributes 20-25 per cent to the total, and the statement that the trades act mainly to store latent heat, while still largely true, must be somewhat modified.

3. Maintenance of the trade winds

Along the mean trade-wind trajectories balance of forces is provided largely by a pressure-gradient force directed downstream acting against friction. The heat-balance calculation just completed raises the question whether we must consider the trade as a driven member of the general circulation sustained from the equatorial zone or perhaps even from the polar westerlies, or whether the maintenance of pressure gradients and

winds can be explained at least partly from more local energy transformations. This subject will be studied from two approaches : (1) we shall compute whether the Pacific trade is an energy exporting or importing circulation branch; and (2) we shall relate heat source and pressure field to determine whether the heat source can be utilized for maintaining the force field.

(*a*) *The trades as an energy exporting circulation*

In view of Table 1 the conservation of energy may be stated to a high degree of accuracy in the form

$$Q = \frac{d}{dt}(\Phi + c_p T). \quad . \quad . \quad . \quad . \quad . \quad (7)$$

For present purposes the change in enthalpy will be divided into the change in internal energy and the work done by pressure forces on the mass considered. Thus,

$$c_p \frac{dT}{dt} = c_v \frac{dT}{dt} + \frac{d}{dt}\left(\frac{p}{\rho}\right), \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and

$$Q - \frac{d}{dt}\left(\frac{p}{\rho}\right) = \frac{d}{dt}(\Phi + c_v T), \quad . \quad . \quad . \quad (9)$$

where the right-hand side now contains the changes of potential and internal energy explicitly. Utilizing the data of Table 1, the computation of the energy change is given in Table 4.

TABLE 4. ENERGY CHANGES ALONG THE TRAJECTORY (unit 10^9 cal/day)

	+	−	+	−
Increase in heat content	8·2			
Work done by pressure force		4·4		
Increase in internal energy			10·8	
Loss of potential energy				7·0
Balance	3·8		3·8	

The Pacific north-east trade gains energy. *It must therefore be regarded as an energy-exporting circulation branch* since for steady state the energy must be transferred to other portions of the globe to help maintain the general circulation there.

It is of interest to compute the energy change also with the assumption made in the first paper, namely that the heat source is zero. The increase in enthalpy must then balance the potential-energy release, hence its value must be 7·0 units instead of 15·2 units. This implies slightly lower temperatures, and also pressures, at the downstream end compared to the actual situation; and both internal energy and work terms will be smaller. The energy transformations for this case are shown in Table 5.

TABLE 5. ENERGY CHANGES ALONG THE TRAJECTORY WITH THE ASSUMPTION OF NO EXPORT OF SENSIBLE HEAT (unit 10^9 cal/day)

	−	+	−
Work done by pressure force	2·0		
Increase in internal energy		5·0	
Loss of potential energy			7·0
Balance	2·0		2·0

When the trade acts as accumulator of latent heat only it cannot maintain itself but is dependent on energy releases in distant portions of the general circulation. The heat acquired by the trade-wind air in the course of its equatorward trajectory therefore becomes important for understanding the general circulation, at least in low latitudes, and probably also for its irregular fluctuations.

(*b*) *The trade as a driving circulation*

Excepting the case of Table 5, the energy transformations do not of themselves permit us to deduce whether the trade is a ' driving ' member of the general circulation. This term, which has no generally accepted meaning, will here be understood to denote a circulation which locally maintains itself and which exports energy. As already demonstrated, the trade exports both sensible and latent heat. It remains to determine whether the circulation is locally self-sustaining; this problem will be approached by considering relations between heat source and pressure gradient.

As seen from Fig. 2, the heat source is strongest near the ground and decreases upwards. Above 800 mb the trajectories are parallel to the isentropes; hence the heat source is confined to the air below 800 mb. Similarly, the horizontal-pressure drop along the section (cf. I) is largest at the surface (5 mb) and becomes zero at 2 km (800 mb). Thus both heat source and pressure gradient vanish at the same altitude, a suggestive coincidence. For the purpose of relating these two quantities we can restrict the consideration to the layer below 800 mb which consists mainly of the subcloud and cloud layers.

From Eq. (2)

$$Q \simeq c_p \frac{d\theta}{dt} \simeq c_p \left(v \frac{\partial \theta}{\partial s} + w \frac{\partial \theta}{\partial z} \right). \qquad (10)$$

Inspecting Fig. 2, the vertical gradient of potential temperature is zero in the subcloud layer. In the cloud layer the horizontal advection of potential temperature exceeds the vertical advection by an order of magnitude as can be shown using the profiles of horizontal and vertical wind speed published in (I). Therefore the vertical advection may be omitted from Eq. (10) so that

$$Q = c_p v \frac{\partial \theta}{\partial s},$$

Integrating over the section and denoting vertical averages by a bar,

$$\frac{1}{c_p} \int\int \frac{Q}{v} ds\, dz = (\bar{\theta}_d - \bar{\theta}_u) D, \qquad (11)$$

where D is the depth of the layer with heat source and d and u denote downstream and upstream respectively.

Introduction of Poisson's equation into the hydrostatic formula yields

$$p_0{}^{\kappa} - p^{\kappa} = \frac{gD}{c_p} \frac{1{,}000^{\kappa}}{\bar{\theta}}, \qquad (12)$$

where p and p_0 are the pressures at top and bottom of the layer of depth D. Differentiating Eq. (12) with respect to s (symbol δ),

$$\delta p_0 = \left(\frac{p}{p_0} \right)^{\kappa-1} \delta p - \frac{gD\, 1{,}000^{\kappa}}{R^* \bar{\theta}^2 p_0{}^{\kappa-1}} \delta \bar{\theta}.$$

nce

$$\delta\bar{\theta} = \bar{\theta}_d - \bar{\theta}_u,$$

$$\delta p_0 = \left(\frac{p}{p_0}\right)^{\kappa-1} \delta p - K \iint \frac{Q}{v}\, ds\, dz, \quad . \quad . \quad . \quad (13)$$

here

$$K = \frac{g\ 1{,}000^{\kappa}}{c_p R^* \bar{\theta}^2 p_0^{\kappa-1}} \simeq 1{\cdot}4 \times 10^{-2}\ \mathrm{g^2/cm^2\ sec^2\ cal.}$$

q. (13) establishes an explicit relation between the surface-pressure drop, the heat bsorbed over the section and the wind speed, provided δp, the pressure gradient imposed y extraneous circulations at the top of the heat source, vanishes. Now we have already een from observations that this is the case. Thus,

$$\delta p_0 = - K \iint \frac{Q}{v}\, ds\, dz. \quad . \quad . \quad . \quad . \quad . \quad (14)$$

All quantities are known; calculation shows that the correct surface pressure drop indeed obtained from the right-hand side of the equation. However, in order to make he equation fully acceptable, an argument should be given which demonstrates that δp ust vanish. At present this cannot be done quantitatively but the following qualitative ypothesis is presented.

The heat transmitted from ocean to atmosphere is distributed upward through the loud layer by convection. In the mean, the upper limit of this transport is situated near he level at which the atmosphere is in equilibrium with the underlying surface, i.e. the ressure at which air rising from the surface as an isolated parcel will attain the environ- nent temperature. This limit ensures that the heat source remains mostly restricted to he subcloud and cloud layers as seen from Fig. 2. In addition, a vertical distribution of riction is introduced which is quite different from that often assumed, though not always bserved, in higher latitudes.

On account of the turbulence in the cloud layer a ' gradient wind level ' is not found t 2,000-3,000 ft altitude. As indicated earlier, the balance of forces in the turbulent ayer is essentially given by

$$\frac{\partial p}{\partial s} - \frac{\partial \tau}{\partial z} = 0. \quad . \quad . \quad . \quad . \quad (15)$$

Above the top of this layer the balance of forces must change drastically since, in accord vith expectations, the frictional drag vanishes almost entirely (cf. I, Table 3). Here the omplete first equation of motion (Eq. 4) becomes applicable. Only the large-scale eddy tresses $\rho v \frac{\partial v}{\partial s}$ and $\rho w \frac{\partial v}{\partial z}$ are available to balance any large cross-isobar flow. There is very indication that the magnitude of these stresses is small (I, Table 3) and that the vhole system is not strongly accelerated. Thus at the top of the turbulent layer the alance of forces is similar to that found farther poleward in broad currents with little cceleration. The flow is laminar and parallel to the isobars, therefore is quasi-geostrophic. The influence of the heat source on the pressure field *must be confined to the turbulent ayer itself* in which the heat accumulates. *Hence the heat source performs a dual role – roduction of downstream warming and maintenance of vertical turbulence in a layer of limited hickness. The coincident requirements on the balance of forces and on the mass distribution rising from this dual role are such as to permit direct utilization of the heat gained to maintain he trades.*

The preceding deductions depend specifically on two characteristics of the region studied : existence of a trade inversion, and a vertical wind profile with a curvature in the turbulent layer such that turbulence must act to retard the flow (Fig. 3). This kind of profile is typical of those portions of the trade wind belts where the meridional temperature gradient is directed poleward. The geostrophic wind is strongest at the ground and decreases throughout the trade wind layer. On account of ground friction however the maximum wind is situated some distance above the ground and the vertical wind profile assumes the shape depicted in Fig. 3. Thus the conclusions drawn cannot be readily applied to the tropical and equatorial regions in their entirety. A separate assessment of the role of the heat source must be made for those regions in which instability extends through a deep layer and where the easterlies do not decrease, or perhaps even increase, with height.

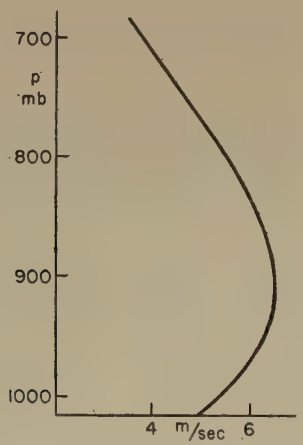

Figure 3. Mean vertical distribution of wind speed (I)

It is finally of interest to consider the maintenance of the trades, assuming no heat source. Then the last term of Eq. (13) would be zero and $\delta p_0 = \left(\frac{p_0}{p}\right)^{\kappa-1} \delta p$. Using $\delta p_0 = 5$ mb, $p = 800$ mb and $p_0 = 1{,}000$ mb, $\delta p = 4{\cdot}3$ mb. A large downstream-pressure drop would still exist at the top of the turbulent layer. This pressure drop may be interpreted as the means by which distant circulations would maintain the trades which, as shown earlier, become an energy-consuming branch of the general circulation when there is no heat source. If large accelerations are to be avoided above the turbulent layer, the wind must veer sharply with height in the trade inversion.

Conclusions

This report has shown that the Pacific north-east trade is a source of sensible as well as of latent heat for the general circulation. Latent heat contributes about 75 per cent, sensible heat 25 per cent, to the total heat export to other parts of the world. The trade current gains and exports energy. Under the specific conditions that prevail with regard to the vertical wind and temperature structure, the heat source is utilized directly to maintain the trade. We may therefore consider the flow in the area studied as a driving branch of the general circulation.

The results suggest extension of the present inquiry in several directions; Malkus 956) has already taken up some of these. The calculations performed may be applied other portions of the tropical and equatorial regions and to the low-latitude meridional rculation cell as a whole. Since the local heat source within a trade has turned out to e an important link in maintaining the current, one may consider the role of varying eat source intensity and wind speed on fluctuations of the general circulation, perhaps llowing the outline of a tropical ' index cycle ' suggested by Riehl (1954).

One of the striking features of the trade is the lack of turning of wind with height rough the inversion, noted early by Ficker (1936) in the Atlantic. Since the depth of ie turbulent layer is much greater in the trades than in the stable atmosphere of the mperate zone, one cannot assume in any derivation of the vertical wind profile that the obars are independent of height in the friction layer. In the trade section it is warmer the downstream than at the upstream end. Since the pressure decreases from right to ft across the section looking downstream, the isobars will rotate counter-clockwise with eight under the influence of the temperature field and thus assume more and more the irection of the surface wind. The foregoing calculations contain a suggestion that under ertain conditions the flow should be two-dimensional in a driving-circulation branch. Considering also the results of Sheppard *et al.* (Sheppard, Charnock and Francis 1952, heppard and Omar 1952) it appears evident that the theory of the wind structure in riction layers is due for revision.

Acknowledgments

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551.571.

Water-vapour flux and flux-divergence over southern England: summer 1954

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SUMMARY

Average zonal (eastward) and meridional (northward) transfers of water vapour for the period June August 1954 are computed at four aerological stations in southern England – Liverpool, Hemsby, Crawley and Camborne – at levels corresponding to the earth's surface, 950, 900, 850, 800, 750, 650, 550, 450, and 350 mb.

In the zonal direction average transfers are eastwards and reach a maximum of 400-500 g (cm 100 mb sec)$^{-1}$ below 900 mb. In the meridional direction both northward and southward transfers occur and their vertical distribution varies from station to station. Largest meridional transfers are of the order of 100 g (cm 100 mb sec)$^{-1}$ and normally occur below 800 mb. Both zonal and meridional transfers become small near 350 mb. Vertically integrated eastward and northward transfers are respectively: Liverpool 1461, − 13; Hemsby 1541, 226; Crawley 1701, 132; Camborne 1781, − 42 g (cm sec)$^{-1}$.

Average zonal eddy-transfers are normally eastward and amount to only a small percentage of the total zonal transfer. Average meridional eddy-transfers are northward, amount to a large percentage of the total meridional transfer and in some cases are oppositely directed. Vertically integrated eastward and northward eddy-transfers are respectively: Liverpool 63, 73; Hemsby 81, 58; Crawley 97, 27; Camborne 113, 125 g (cm sec)$^{-1}$. Although directed roughly from high to low values of average specific humidity, average eddy-transfer vectors at a given (geometric or pressure) level bear little quantitative relation to average (horizontal or isobaric) specific humidity gradients at the same level.

Assuming the above transfers to vary linearly between stations, computation of the net horizontal outflow of water vapour from the region shows marked convergence of water vapour from the surface to 850 mb with divergence above this. For the three months' period this amounts to a net inflow of 31 g cm^{-2} at low levels and a net outflow of 23 g cm^{-2} at higher levels. Increase in storage amounts to approximately 1 g cm^{-2} thus leaving a surplus of precipitation over evaporation of approximately 7 g cm^{-2}. Estimation of precipitation from isohyetal charts and evapotranspiration by empirical methods yields a corresponding surplus of 6 g cm^{-2}.

Computation of the large-scale vertical transfer of water vapour (using instantaneous kinematically-computed vertical velocities) at 900, 800, 700 and 600 mb showed that at these levels large-scale vertical eddy-transfer of water vapour is negligibly small. Using mean vertical velocities and specific humidity averaged over the area at 950, 850, 750, 650, 550 and 450 mb, computations yield large-scale upward transfers of water vapour for the three months' period amounting to 17, 23, 13, 6, 2, 0 g cm^{-2} respectively at the above levels. This flux is shown to be inadequate for water-balance requirements and it is concluded that a considerable part of the upward transfer is carried by turbulent and convective systems of essentially smaller scales.

1. INTRODUCTION

The transfer of water vapour in the earth's atmosphere is known to play an important part in many atmospheric processes. Thus the horizontal transfer of latent heat between latitudes forms an important part of the atmospheric energy balance while the transfer of latent heat in the vertical is closely associated with the vertical distribution of net atmospheric radiational heating or cooling.

Atmospheric transfer of water vapour is also important in the working of the hydrologic cycle. In order to obtain a more complete understanding of this cycle and to gain a clearer appreciation of the water economies of both continental and oceanic areas it is necessary to have quantitative data on the transfer of water vapour by the atmosphere. It is a striking fact that over the years our increased quantitative knowledge of the precipitation and evaporation processes involved in the hydrologic cycle has not nearly been matched by corresponding knowledge of the atmospheric water-vapour transfer fields which are essential to its operation. Moreover, quantitative assessment of the role of the atmosphere in the water-balance of a particular area may often afford a valuable check on the adequacy of hydrologic data obtained in other ways.

There are thus good reasons for making extensive observational studies of the transfer f water vapour in the atmosphere. Nevertheless, up to the present, such studies have een remarkably few in number and have been mainly global (Starr and White 1954) or ontinental (Benton and Estoque 1954) in scope. These investigations have proved of reat interest but it nevertheless seems probable that many of the important and charac- ristic features of the water-balance of a smaller area would be lost when very large areas uch as major continents or latitudinal zones are considered. It therefore seems very esirable to make a detailed study of the water-balance of a relatively small area and articularly of one for which adequate aerological observations are available. This paper epresents a preliminary attempt at such a study. Its main purposes are :

(i) To give quantitative data on the horizontal transfer of water vapour for a particular period at four aerological stations in southern England.

(ii) To determine the atmospheric levels at which significant transfer takes place and to investigate the part played by large-scale atmospheric eddy-motion in such transfer.

(iii) To compute at each level the average divergence of the horizontal water-vapour transfer vector field over southern England and to investigate the distribution of this quantity with height.

(iv) To investigate the possibility of combining the results of such computations with precipitation measurements to give an independent estimate of the water loss from the area by evapotranspiration.

(v) To compute the large-scale vertical transfer of water vapour over the area and to assess, by comparing this transfer with that required by balance considerations, the contribution to vertical transfer made by turbulent and convective systems of smaller scales.

2. Computation of horizontal water-vapour transfer

(*a*) *Total transfer*

Average water-vapour transfers for the summer period June-August 1954 have been omputed at four aerological stations in southern England – Liverpool (53°28′N 02°55′W), Hemsby (52°41′N 01°41′E), Crawley (51°05′N 00°13′W) and Camborne (50°13′N 5°19′W) at mean levels corresponding respectively to the earth's surface, 950, 900, 850, 00, 750, 650, 550, 450 and 350 mb. At each of these levels the average zonal (eastward) nd meridional (northward) fluxes of water vapour have been evaluated by means of the llowing expression for the water-vapour transfer vector **w**

$$\mathbf{w} = \frac{1}{gT}\int_0^T q\mathbf{c}\,dt \approx \frac{1}{gN}\sum_{i=1}^{i=N}(q_i\,\mathbf{c}_i)$$

In these expressions N is the number of observations in the time interval T, q is the pecific humidity, t the time, g the acceleration of gravity, $\mathbf{c}$ the horizontal wind vector, while i denotes the individual observations over which the summation Σ is to be per- ormed. The average zonal and meridional water-vapour transfers at each level are onveniently expressed in units of g (cm 100 mb sec)$^{-1}$ and the vertically integrated ransfers in units of g (cm sec)$^{-1}$.

All available data were used in the computations. These consisted of radio-sonde nd radar-wind observations taken regularly at 02 hr and 14 hr GMT each day during he period under investigation. During this period both radio-sonde and radar-wind

observations reached high levels with great regularity so that only a very small percentage of the observations was missing. These few missing values were estimated from wind observations at other hours (08 hr and 20 hr GMT) and from humidity values obtained by interpolation between soundings. These interpolations were assisted by reference to the published upper level and surface synoptic charts. All these data were taken from the *Daily Aerological Record* of the Meteorological Office, London. At the earth's surface it was possible to evaluate the above time-integral more accurately by using four observations daily. This was done by using surface wind data taken at the aerological stations at 08 hr and 20 hr GMT together with observations of surface dew-point at 06 hr and 18 hr GMT at stations sufficiently close to the aerological stations being used for the main transfer studies. For this purpose Squires Gate, Gorleston, Tangmere and St. Eval were considered as representative of Liverpool, Hemsby, Crawley and Camborne respectively as far as surface dew-point was concerned. The wind data were taken from the *Daily Aerological Record* and the dew-point data from the *Daily Weather Report* both of the Meteorological Office, London.

Values of water-vapour transfer obtained in this manner were used to compute, for each of the four stations, the zonal and meridional components of the vertically integrated transfer vector $\mathbf{W}$ given by the relation

$$\mathbf{W} = \int_0^{p_0} \mathbf{w}\, dp$$

The integrated transfer was evaluated numerically by applying the trapezoidal rule to the curves of water-vapour transfer as a function of pressure from the earth's surface (as represented by the mean station-level pressure p_0) to the 350 mb level. Above this level only a small amount of water-vapour transfer occurred and this was evaluated by linear extrapolation of the transfer values to zero. At the higher levels all the above curves showed the appropriate tendency towards zero transfer and the above procedure normally resulted in a vanishing of the water-vapour flux at a level between 300 and 250 mb. The exact level at which the flux was assumed to vanish was found to make no significant difference to the evaluated integrated flux.

(*b*) *Advective transfer*

At any particular level the total horizontal water-vapour transfer vector may be thought of as consisting of two parts. The first of these, here called the advective transfer, is the transfer associated with the time averages of the horizontal wind vector and specific humidity at the particular level concerned. Thus at any level the advective transfer $\mathbf{w}_M$ is given by

$$\mathbf{w}_M = \frac{1}{g}\,\bar{q}\,\bar{\mathbf{c}}$$

while the corresponding vertically integrated advective transfer $\mathbf{W}_M$ is given by

$$\mathbf{W}_M = \int_0^{p_0} \mathbf{w}_M\, dp$$

In the above relationships the time averages of q and $\mathbf{c}$ over the time interval T are defined by

$$\bar{q} = \frac{1}{T}\int q\, dt \approx \frac{1}{N}\sum_{i=1}^{i=N} (q_i) \qquad \bar{\mathbf{c}} = \frac{1}{T}\int \mathbf{c}\, dt \approx \frac{1}{N}\sum_{i=1}^{i=N} (\mathbf{c}_i)$$

The observational data used here were the same as those used for the evaluations of total water-vapour transfer and similar computational procedures were followed. In

rticular the same procedures for evaluating the vertically integrated advective flux om the earth's surface to 350 mb and for estimating the advective flux above this level oved satisfactory.

(c) *Eddy transfer*

The second part of the total transfer is the residual transfer $\mathbf{w}_E$, here called the dy-transfer, associated with the instantaneous departures at any particular level of q d $\mathbf{c}$ from their respective means. Thus at any level the eddy-transfer $\mathbf{w}_E$ is given by

$$\mathbf{w}_E = \mathbf{w} - \mathbf{w}_M$$

hile the corresponding vertically integrated eddy-transfer $\mathbf{W}_E$ is given by

$$\mathbf{W}_E = \mathbf{W} - \mathbf{W}_M$$

These eddy-transfers are mainly associated with the passage over the station of rge-scale migratory disturbances such as are exemplified by the closed cyclonic and ticyclonic systems of the surface synoptic chart or the large-scale trough and ridge tterns of the upper atmospheric levels. They do not include the water-vapour transfer sociated with more or less permanent features of the atmospheric circulation such as e exemplified by the large geographically fixed semipermanent surface cyclones and ticyclones or local meridional and zonal cells.

ABLE 1. AVERAGE TOTAL, ADVECTIVE AND EDDY TRANSFERS OF WATER VAPOUR, SOUTHERN ENGLAND, JUNE-UGUST 1954. TRANSFERS AT EACH LEVEL g (cm 100 mb sec)$^{-1}$, VERTICALLY INTEGRATED TRANSFERS g (cm sec)$^{-1}$

ation	Level (mb)	Surface	950	900	850	800	750	650	550	450	350	Vertical integral	Mean station level pressure (mb)
						EASTWARD							
iverpool	Total	+212	+381	+386	+351	+333	+289	+206	+127	+ 80	+ 35	+1461	1009
	Advective	+207	+372	+378	+340	+322	+280	+193	+120	+ 69	+ 29	+1398	
	Eddy	+ 5	+ 9	+ 8	+ 11	+ 11	+ 9	+ 13	+ 7	+ 11	+ 6	+ 63	
emsby	Total	+176	+401	+406	+381	+341	+293	+220	+152	+ 80	+ 35	+1541	1010
	Advective	+186	+403	+406	+375	+335	+280	+195	+125	+ 68	+ 30	+1460	
	Eddy	− 10	− 2	0	+ 6	+ 6	+ 13	+ 25	+ 27	+ 12	+ 5	+ 81	
rawley	Total	+165	+486	+453	+429	+387	+339	+241	+164	+ 91	+ 44	+1701	996
	Advective	+164	+474	+443	+410	+364	+316	+222	+148	+ 83	+ 40	+1604	
	Eddy	+ 1	+ 12	+ 10	+ 19	+ 23	+ 23	+ 19	+ 16	+ 8	+ 4	+ 97	
amborne	Total	+256	+527	+483	+432	+382	+320	+233	+172	+107	+ 48	+1781	1004
	Advective	+254	+507	+458	+396	+349	+290	+217	+163	+102	+ 46	+1668	
	Eddy	+ 2	+ 20	+ 25	+ 36	+ 33	+ 30	+ 16	+ 9	+ 5	+ 2	+ 113	
						NORTHWARD							
iverpool	Total	− 5	− 21	+ 1	+ 14	+ 10	− 1	− 8	− 3	− 3	− 3	− 13	1009
	Advective	− 11	− 38	− 21	− 8	− 8	− 15	− 17	− 10	− 7	− 4	− 86	
	Eddy	+ 6	+ 17	+ 22	+ 22	+ 18	+ 14	+ 9	+ 7	+ 4	+ 1	+ 73	
emsby	Total	+ 63	+ 41	+ 41	+ 60	+ 58	+ 58	+ 37	+ 17	+ 5	+ 2	+ 226	1010
	Advective	+ 57	+ 23	+ 30	+ 41	+ 46	+ 43	+ 26	+ 13	+ 6	+ 1	+ 168	
	Eddy	+ 6	+ 18	+ 11	+ 19	+ 12	+ 15	+ 11	+ 4	− 1	+ 1	+ 58	
rawley	Total	+ 89	+120	+ 68	+ 53	+ 23	+ 4	− 4	− 5	− 7	− 5	+ 132	996
	Advective	+ 78	+ 98	+ 52	+ 44	+ 24	+ 4	− 5	− 5	− 7	− 5	+ 105	
	Eddy	+ 11	+ 22	+ 16	+ 9	− 1	0	+ 1	0	0	0	+ 27	
amborne	Total	+ 45	+ 53	0	− 20	− 20	− 21	− 9	− 16	− 15	− 10	− 42	1004
	Advective	+ 32	+ 13	− 41	− 52	− 46	− 39	− 26	− 27	− 20	− 10	− 167	
	Eddy	+ 13	+ 40	+ 41	+ 32	+ 26	+ 18	+ 17	+ 11	+ 5	0	+ 125	

3. Water-vapour transfer data

For the period June-August 1954, at each of the levels evaluated, the average zonal and meridional total, advective and eddy transfers of water vapour for Liverpool, Hemsby, Crawley and Camborne are given in Table 1, together with the vertically integrated average transfers at each station. Transfers at the earth's surface are those computed using four observations daily while those at 950 mb and above are computed from data taken only twice daily. Total transfer data for each station are plotted against a linear pressure scale in Fig. 1 and illustrate the general character of the vertical distribution of the zonal and meridional transfers.

The curves for zonal transfer have a very similar character, all showing a well-marked maximum of flux below the 900 mb level and an approximately linear decrease of the flux with decreasing pressure above this maximum. At the northern stations, Liverpool and Hemsby, this maximum appears to be close to the 925 mb level while at the southern stations, Crawley and Camborne, the maximum seems to be near to or even below the 950 mb level. Vertically integrated transfers are greater at the southern stations and significant vapour transfer extends to somewhat higher levels. Nevertheless it would seem that at each station only about 1 per cent of the vertically integrated flux occurs above the 350 mb level.

The curves for meridional transfer show on the whole much smaller values and exhibit considerable differences in character from station to station. Vertically integrated transfers are northward at the eastern stations Hemsby and Crawley and southward at the western stations Liverpool and Camborne. The flux at Crawley appears somewhat anomalous in that almost all of the transfer is accomplished below the 750 mb level, but in other respects the distribution resembles that at Hemsby. The distributions at Liverpool and Camborne are similar in as much as the southward character of the integrated flux is obtained mainly from the upper levels, but at lower levels the distributions have little in common. On a percentual basis meridional transfers do not decrease with height as rapidly as do zonal transfers, the percentage of the integrated flux occurring above 350 mb at Liverpool, Hemsby, Crawley and Camborne being estimated at 4, 1, 2 and 8 per cent respectively.

The relative importance of the contributions to the total transfer made by advective and eddy components respectively may be inferred from the data given in Table 1. From this table it appears that the absolute values of the eddy-transfers in the zonal and meridional directions are of about the same order of magnitude but that on a percentual basis eddy-transfers are small in the zonal direction and large in the meridional direction.

At all four stations and at all levels evaluated the zonal eddy-transfers are directed eastward, except for relatively small westward transfers at the surface and 950 mb levels at Hemsby. This is no doubt due to some local peculiarity or to the frequent occurrence of sea-breezes on this part of the coast. Meridional eddy-transfers are directed northward except in a few cases where the eddy-transfer is nearly zero. In such cases the southward direction has little significance. It is striking that for Liverpool and Camborne at almost every level the advective and total transfers are southward, while at all levels eddy-transfer is northward.

Eddy-transfer data for each station are plotted against a linear pressure scale in Fig. 2 and illustrate the general character of the vertical distribution of eddy-transfer in the zonal and meridional directions. These curves show more variation from station to station than was the case with the curves for total transfer.

At Crawley and Camborne zonal eddy-transfers are small near the surface and reach a maximum near the 850-800 mb level. At Hemsby, however, the maximum eddy-transfer is not reached until the 550 mb level while at Liverpool there is no definite

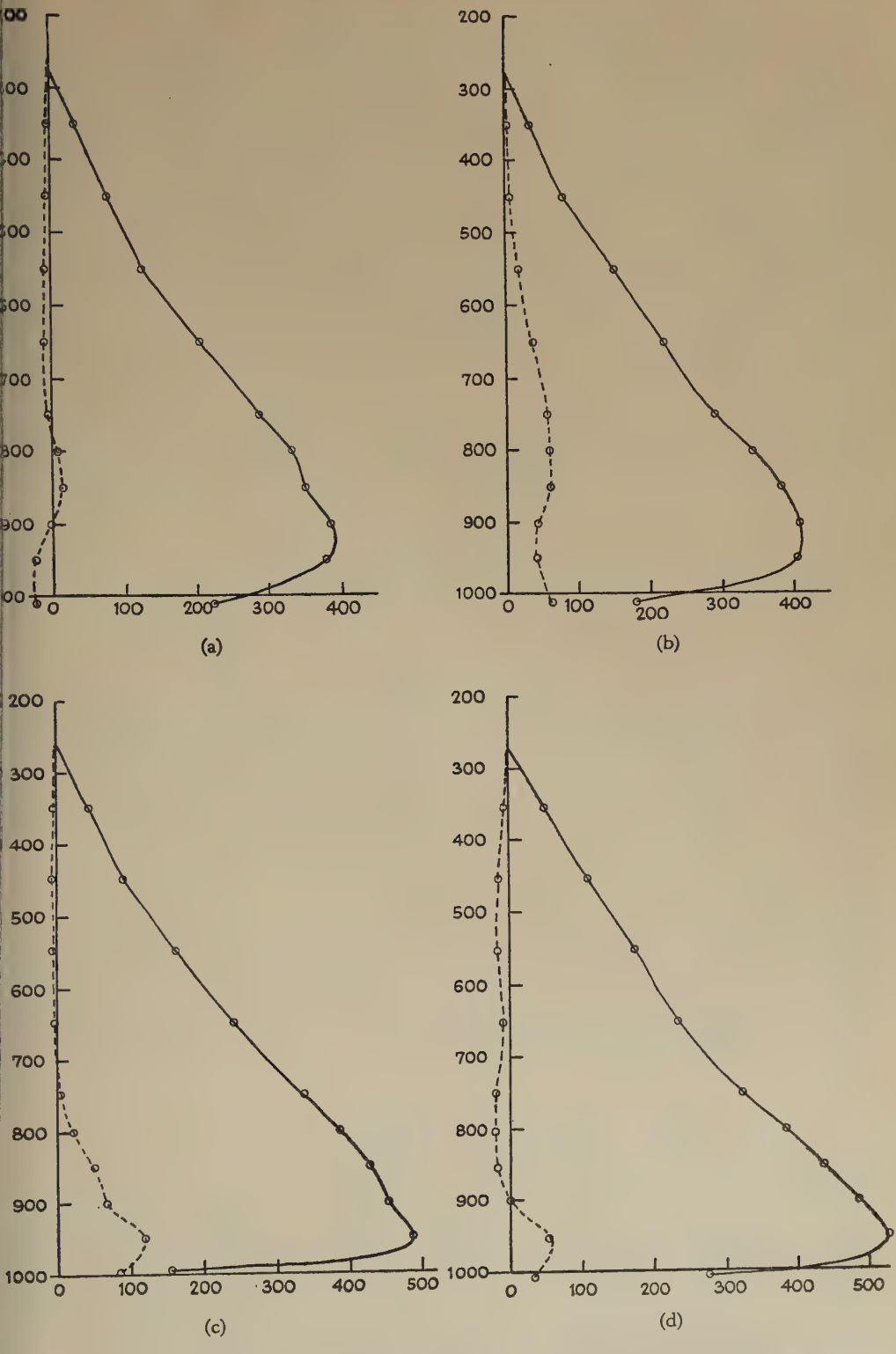

Figure 1. Vertical distribution of average zonal (full) and meridional (dashed) total transfer of water vapour at (a) Liverpool, (b) Hemsby, (c) Crawley, (d) Camborne. Vertical unit : mb; horizontal unit : g (cm 100 mb sec)$^{-1}$. Extrapolated above 350 mb.

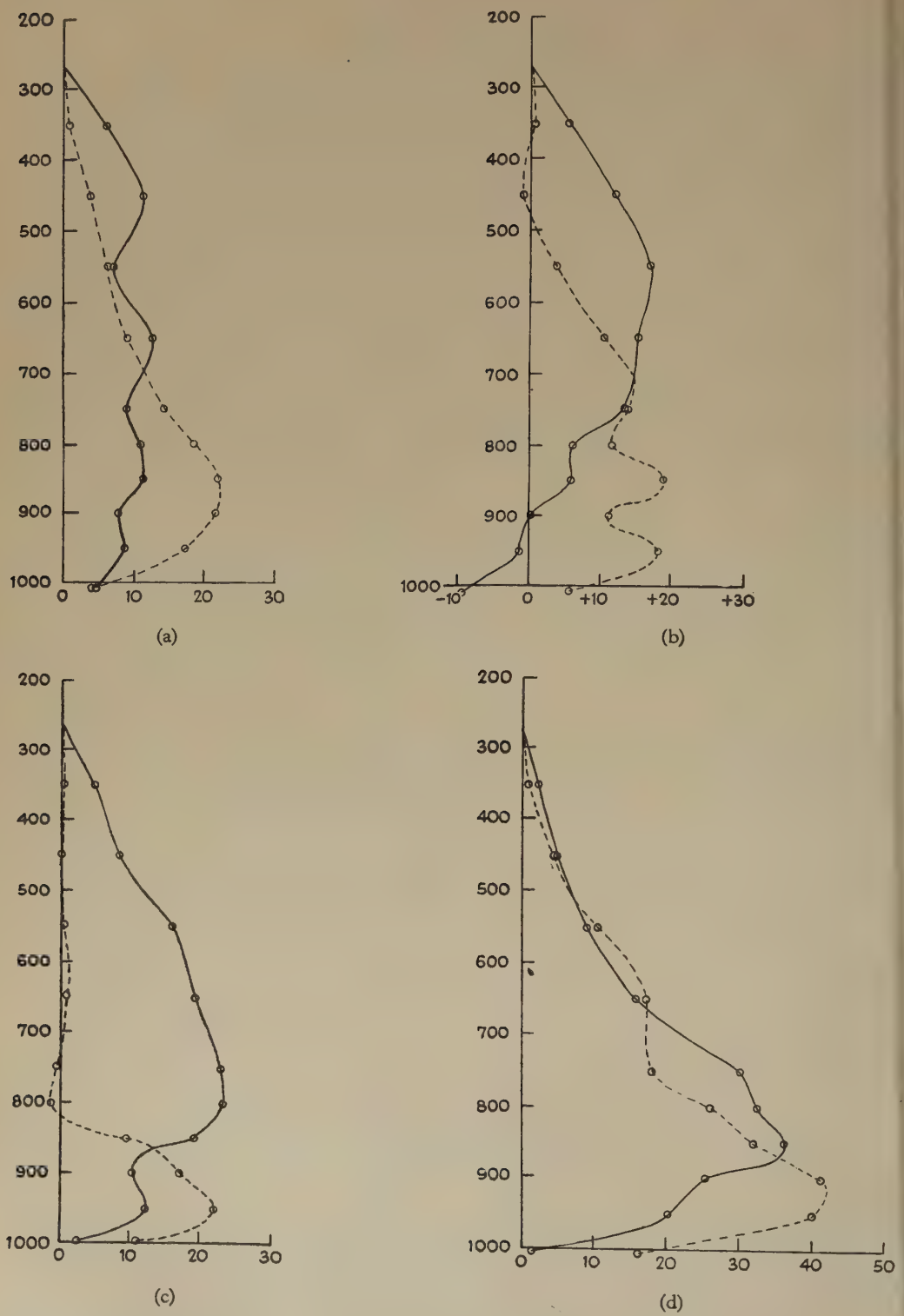

Figure 2. Vertical distribution of average zonal (full) and meridional (dashed) eddy-transfer of water vapour at (a) Liverpool, (b) Hemsby, (c) Crawley, (d) Camborne. Vertical unit : mb; horizontal unit : g (cm 100 mb sec)$^{-1}$. Extrapolated above 350 mb.

ximum but a broad layer from 950-400 mb in which the zonal eddy-transfer remains atively constant. For meridional eddy-transfers Liverpool and Camborne show rather nilar vertical distributions with the maximum eddy-flux at 900-850 mb. At Hemsby ere is a rather broad layer of maximum but rather variable eddy-flux from 950 to 700 mb ile at Crawley, after a pronounced maximum at 950 mb, the eddy-flux rapidly decreases near zero at 800 mb and remains essentially zero to the highest levels.

From the zonal and meridional eddy-transfer data given in Table 1, eddy-transfer ctors were calculated for the four stations at each of the pressure levels under con-leration. These were plotted on charts showing the distribution of the average specific midity for the three months' period at the pressure levels in question. Similar charts re also prepared showing approximate eddy-transfer vectors at horizontal geometric vels near the pressure surfaces under consideration and the approximate distribution the average specific humidity at these geometric levels. It was apparent that although dy-transfer vectors were in most cases directed approximately from high to low values the average specific humidity there was little quantitative relation between average obaric or geometric) gradients of specific humidity and the corresponding eddy-transfer ctors. It appears therefore that mixing-length formalism would be of little use in timating large-scale turbulent transfer of atmospheric water vapour. This result is accord with those obtained previously (Priestley 1949) for the large-scale turbulent nsfer of heat and momentum in the meridional direction over Larkhill in southern gland.

4. Accuracy of computed transfer data

The accuracy of the computed average water-vapour transfer data has been con-dered at four representative levels – 950, 750, 550 and 350 mb – and the discussion mmarized under the following headings (a) wind errors, (b) instrumental errors in midity, (c) lag errors in humidity, (d) total error in humidity, (e) resulting error in nsfer, (f) sampling errors in transfer, (g) total error in transfer, (h) error in vertically tegrated transfer.

(*a*) *Wind errors*

The errors inherent in the radar-wind observations made at British land-based ations have been discussed by Bannon (1948). From this discussion it may be con-uded that there are no significant systematic errors in the wind observations and that e standard vector error of the observed wind is of the order of 1 kt at the 950 and 0 mb levels, between 1 and 2 kt at the 550 mb level and between 2 and 3 kt at the 0 mb level.

(*b*) *Instrumental errors in humidity*

The random instrumental errors inherent in observations of relative humidity made ith the British Kew radio-sonde have been studied by Dymond (1947) and Malet (1952). n evaluating the results of the Payerne radiosonde comparisons, Nyberg (1952) quotes n investigation by Harrison and Blackledge to the effect that the different types of umidity element show only small systematic differences. From these studies it may e concluded that there are no important systematic instrumental errors in humidity bservations taken with the Kew radiosonde and that the standard instrumental error f these observations is approximately 1 unit of relative humidity (per cent) at 950 mb, at 750 mb, 4 at 550 mb and 4 at 350 mb.

(c) *Lag errors in humidity*

Errors due to lag were evaluated at each station by applying Väisälä's (1952, 1953) method to a random sample of 100 observations at each of the selected levels. In accordance with the work of Glückauf (1947), Gondet (1949), Petit (1950) and Schulze (1952), a lag coefficient of 20 sec was assumed at 0°C for an actual (and indicated) relative humidity of 100 per cent. Values for the lag coefficient at other temperatures and indicated relative humidities were calculated from Väisälä's formula.

The distributions of these errors, considered as corrections to the observed humidity, were slightly skew but could be usefully characterized by mean and standard deviation. Average values of these quantities over the four stations were respectively — 0·2 and 0·6 units at 950 mb, — 1·2 and 2·1 units at 750 mb, — 2·2 and 5·3 units at 550 mb, — 3·3 and 7·1 units at 350 mb. Thus, on the average, the use of humidity observations uncorrected for lag introduces only a very small systematic error at the 950 mb level but leads to a systematic overestimation of humidity at the 750, 550 and 350 mb levels amounting to 1, 2 and 3 units respectively. In addition, the use of such humidities introduces random errors in relative humidity whose root-mean-square values are approximately 1, 2, 5 and 7 units respectively at the 950, 750, 550 and 350 mb levels.

(d) *Total error in humidity*

Random errors due to lag and random instrumental errors due to other causes are additive in mean square and result in a combined standard error of 1 unit (per cent) of relative humidity at 950 mb, 5 units at 750 mb, 7 units at 550 mb and 8 units at 350 mb. Systematic errors remain as in (c).

(e) *Resulting error in transfer*

Systematic humidity errors indicate that the magnitudes of the average zonal transfers at the 950, 750, 550 and 350 mb levels are respectively overestimated by approximately 1, 6, 5 and 2 g (cm 100 mb sec)$^{-1}$ at Liverpool, 1, 6, 5, and 2 g (cm 100 mb sec)$^{-1}$ at Hemsby, 1, 7, 6 and 3 g (cm 100 mb sec)$^{-1}$ at Crawley and by 1, 7, 7 and 3 g (cm 100 mb sec)$^{-1}$ at Camborne. The magnitudes of the average meridional transfers are similarly overestimated but this overestimate amounts to as much as 1 g (cm 100 mb sec)$^{-1}$ only at the 750 mb level at Hemsby and the 750 and 550 mb levels at Camborne.

Random humidity errors indicate that the standard vector errors in the average transfer vectors, at the 950, 750, 550 and 350 mb levels are respectively 3, 3, 2 and 1 g (cm 100 mb sec)$^{-1}$ at Liverpool and Hemsby and 3, 4, 2 and 1 g (cm 100 mb sec)$^{-1}$ at Crawley and Camborne.

(f) *Sampling errors in transfer*

These arise through the use of the arithmetic mean of two (at the surface four) observations daily to represent the required time-integrated average transfer. The standard sampling error ϵ has been calculated from the formula (Cochran 1946, Quenouille 1949), $\epsilon^2 = \frac{b^2}{n}\left[\frac{1+\rho}{1-\rho} + \frac{2}{\ln\rho}\right]$ where b is the standard deviation and ρ the 12 hr-lag autocorrelation coefficient (assumed exponential) of the individual zonal and meridional transfers. Exponential autocorrelation coefficients are well-established for wind data (Durst 1954).

Average values for two stations (Liverpool and Crawley) of the 12 hr-lag autocorrelation coefficients for zonal and meridional transfers were respectively 0·75 and 0·71 at 0 mb, 0·74 and 0·69 at 750 mb, 0·69 and 0·58 at 550 mb, 0·65 and 0·54 at 350 mb, while the corresponding average standard deviations were respectively 361 and 320, 278 and 247, 148 and 136, 42 and 38 g (cm 100 mb sec)$^{-1}$ at these levels.

Use of the formula quoted above discloses standard vector errors in the average transfer vectors amounting at each station to 8, 6, 4 and 1 g (cm 100 mb sec)$^{-1}$ at 950, 750, 550 and 350 mb respectively. It appears therefore that for two observations daily the sampling error at all except the highest levels is considerably greater than the combined random error due to other causes.

(*g*) *Total error in transfer*

Combining in mean square the random errors previously computed with the sampling errors it is found that final values for the standard vector errors of the computed average water-vapour transfer vectors are at each station approximately 9, 7, 4 and 1 g (cm 100 mb sec)$^{-1}$ respectively at the 950, 750, 550 and 350 mb levels. The systematic errors remain as computed in (*e*).

(*h*) *Error in vertically integrated transfer*

This was computed from the individual errors taken at the centres of layers 100 mb thick extending from 1,000 mb to 300 mb. The errors at 950, 750, 550, and 350 mb were obtained by calculation and those at the remaining levels were obtained by interpolation.

With regard to systematic errors, these are additive level by level and lead to an overestimation of the magnitude of the vertically integrated average zonal transfers amounting to 26 g (cm sec)$^{-1}$ at Liverpool, 27 g (cm sec)$^{-1}$ at Hemsby, 32 g (cm sec)$^{-1}$ at Crawley and 34 g (cm sec)$^{-1}$ at Camborne. Vertically integrated average meridional transfers are similarly overestimated in magnitude by approximately 2 g (cm sec)$^{-1}$ at Liverpool, 3 g (cm sec)$^{-1}$ at Hemsby, 1 g (cm sec)$^{-1}$ at Crawley and 5 g (cm sec)$^{-1}$ at Camborne. With regard to random errors, a detailed investigation shows that the vertical correlation between the total transfer errors at individual levels is very small and that, to the order of accuracy required, the final result is the same as that obtained by assuming vertical independence of the transfer errors. This procedure results in a standard vector error of 16 g (cm sec)$^{-1}$ in the average vertically integrated water-vapour transfer vectors for each of the stations being considered. Expressed as a percentage of the magnitude of the average vertically integrated transfer vectors at each station this error amounts to about 1 per cent.

5. Divergence of water-vapour flux

The total water-vapour transfers set out in Table 1 have been used to compute at each level the average net horizontal outflow (divergence) of water vapour (referred to unit horizontal area and 100 mb vertical pressure interval) which occurred during the period June-August 1954 over the quadrangular area of approximately 9×10^4 (km)^2 delimited by the aerological stations being considered. This divergence was also integrated vertically to give the average total horizontal outflow of water vapour (for unit horizontal area) over the region being considered. In the divergence computations the transfers of water vapour were assumed to vary linearly between aerological stations at the earth's surface and at each isobaric level. In computing the integrated divergence a linear variation of divergence in the vertical was assumed between each pressure level for which data

had been evaluated. The pressure intervals used were thus 50 mb from 950 mb to 750 mb and 100 mb above this. The interval from the earth's surface to the 950 mb level is of variable pressure-thickness from day to day and from place to place but has been assumed to be given by the difference between 950 mb and the mean station pressure for the aerological stations concerned. This mean pressure was 1,005 mb.

This procedure does not take into account the question of how far the presence of mountain barriers or topographical irregularities of only minor extent may affect the flow of moisture into and out of the space under consideration. With regard to this question two extreme hypotheses seem possible. On the first hypothesis it could be supposed that a linear variation of transfer between stations is valid but that the mountain barrier simply blocks out that part of the face and leads to a correspondingly diminished transfer through the face in question. On the second hypothesis it might be supposed that the flow of vapour is considerably increased above and between the topographical irregularities so that the hypothesis of a linear variation of transfer between aerological stations becomes untenable. Nevertheless in this case approximately the same amount of water vapour passes through the face as if the mountain barrier were supposed non-existent and a linear variation of transfer between aerological stations assumed.

Detailed transfer calculations based on topographical profiles along the sides of the area in question readily show that under the first hypothesis the adopted procedure would lead to an overestimate of the total inflow of water vapour across the Camborne-Liverpool face (per unit horizontal area) of approximately 16 g cm^{-2} while the total outflow across the other faces (per unit horizontal area) is overestimated by approximately 2 g cm^{-2}. Thus there would be a spurious convergence of water vapour due to topography of about 14 g cm^{-2}. Although under the first hypothesis there is no doubt regarding the importance of this effect it is very doubtful if this hypothesis ever operates in the atmosphere to any significant degree. Actually, in view of our present knowledge regarding the flow of air over and around mountainous country the second hypothesis seems by far the more probable. For this reason the divergence of water vapour has been computed under essentially the second hypothesis. This procedure will tend, if anything, to underestimate the total outflow of water vapour from the space under consideration.

In the above computations the curvature of the earth's surface has been taken into account in a sufficiently accurate way by allowing for the slight variation of the degree of longitude which occurs over the area. The adequacy of this curvilinear model was tested by computing the horizontal velocity divergence for a northerly wind of 1 m sec^{-1} distributed uniformly over the area. This gave a value of $2{\cdot}0 \times 10^{-7}$ sec^{-1}, which is the same as that given by the usual formula $-\frac{v \tan \phi}{a}$ where v is the poleward wind component, ϕ the latitude (52°N) and a the earth's radius (6,371 km). Computations were also carried out on the assumption that the earth's surface over the area might be considered as a plane. The effect of allowing for the earth's curvature varied from level to level but the overall effect was to increase the net convergence of water vapour over the area by approximately 1 g cm^{-2} for the three months' period under consideration.

The results of these calculations are shown in Table 2 together with the results of similar computations for the divergence of the advective and eddy fluxes. These values are represented graphically as a function of pressure in Fig. 3 and demonstrate the character of the average water-vapour flux divergence over the area during the period under consideration. A strong convergence of water vapour is evident in the surface layers but this rapidly decreases and vanishes just below the 850 mb level. Above this level well-marked divergence exists over a deep atmospheric layer, but this becomes relatively weak above 450 mb. The distribution of total water-vapour flux divergence described

ove may show some features belonging to the general planetary circulation but the stribution agrees well with what might be expected over a land area in summer and is erefore most probably to be regarded as mainly due to a regional circulation between nd and sea.

ABLE 2. NET HORIZONTAL OUTFLOW (POSITIVE) OF WATER VAPOUR g cm^{-2} (100 mb)$^{-1}$ FOR PERIOD JUNE-AUGUST 1954

vel (mb)	1,005	950	900	850	800	750	650	550	450	350
otal	− 34·4	− 30·2	− 12·4	+ 1·3	+ 5·3	+ 9·9	+ 5·7	+ 5·2	+ 1·0	+ 1·1
dvective	− 31·2	− 25·0	− 7·9	+ 2·6	+ 4·1	+ 6·0	+ 2·6	+ 1·4	+ 0·2	+ 0·8
ddy	− 3·2	− 5·2	− 4·5	− 1·3	+ 1·2	+ 3·9	+ 3·1	+ 3·8	+ 0·8	+ 0·3

The divergence of the advective flux is distributed with height in a manner similar to that of the total flux but, except at the 850 mb level (where both values are very small), is always numerically smaller than the total flux divergence. This results in a low level eddy-flux convergence extending to slightly above the 850 mb level with divergence of eddy-flux above this. This divergence becomes insignificant near the 350 mb level. Divergence of all three types of flux reaches a maximum in the lower troposphere near the 750 mb level.

Computation of the vertically integrated total flux divergence (total outflow of water vapour per unit horizontal area for the period considered) results in a total convergence of 31 g cm^{-2} of water vapour in the lower (approximately below 850 mb) atmosphere and a divergence of 23 g cm^{-2} in the upper atmospheric layers. There is thus an apparent surplus over the area of 8 g cm^{-2} of water.

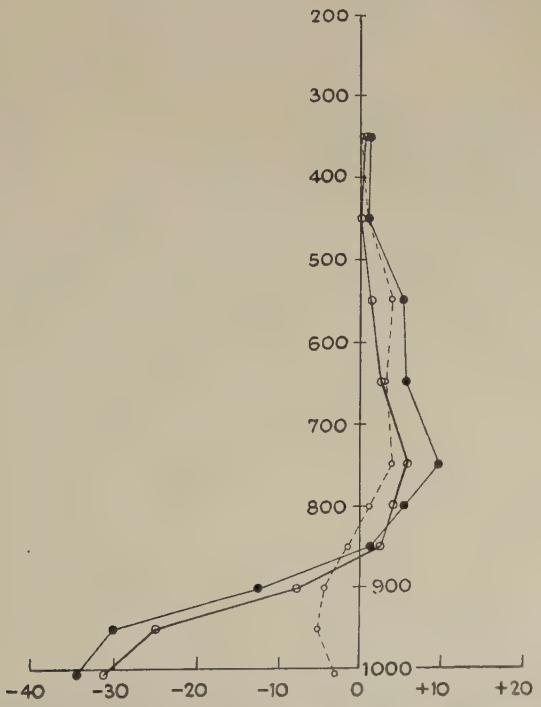

Figure 3. Average horizontal divergence (positive) of total (full circles), advective (open circles) and eddy (dashed) transfer of water vapour. Vertical unit : mb; horizontal unit : g cm^{-2} (100 mb)$^{-1}$.

This estimate assumes no significant difference between the initial and final water-vapour contents of the air. To investigate this point initial and final water-vapour contents were computed by integrating the specific humidity at each of the aerological stations concerned from the mean surface pressure to 300 mb, and averaging over the four stations. Values obtained in this way relate mainly to the boundary of the area and to compensate for this humidity observations from aircraft soundings at Worcester (in the central part of the area) were incorporated in the estimates of initial and final water-vapour contents. These calculations gave the initial water-vapour content as 1·7 g cm^{-2} and the final water-vapour content as 2·6 g cm^{-2}. Thus approximately 1 g cm^{-2} of the above apparent surplus has been stored in the atmosphere over the area leaving an actual surplus of approximately 7 g cm^{-2}. An appeal to commonly accepted orders of magnitude readily shows that in the present context the transfer of water in liquid or solid form is entirely negligible and on this basis the surplus of 7 g cm^{-2} must be accounted for by an excess of precipitation over evapotranspiration of approximately this amount.

6. Accuracy of computed water-vapour flux-divergence

In the previous section computations of the average horizontal divergence of water vapour over a quadrangular area were carried out under the assumption of a linear variation of water-vapour transfer between aerological stations situated at the corners of the area in question. Essentially this amounts only to the less stringent assumption that the average value of the mean water-vapour transfer over a line segment equals the average of the mean transfers at the end-points of the segment. The truth of this assumption depends primarily on the magnitude of the spatial variation of mean water-vapour transfer on a scale smaller than that for which we have direct observational knowledge, and it is thus not possible to give an exact estimate of the error involved in such computations.

It may be expected, however, that in the mean field of water-vapour transfer any systematic departure from the above assumption will show only very small spatial variations over the area concerned and would thus lead to only negligible systematic error in the divergence of the average water-vapour transfer field. On the other hand, on individual occasions, departures from linearity are likely to be more serious and will lead to some degree of random uncertainty in the computed divergence of water vapour averaged over the three months' period considered. Some measure of this uncertainty may be obtained by referring to the estimates made by Sheppard (1949) of the random errors in wind divergence which may arise owing to the non-fulfilment of the assumption of a linear variation of wind between aerological stations. For comparable distances between aerological stations, but for triangles of roughly half the area of the region used here, Sheppard (1949) gives the maximum error in instantaneous wind divergence due to non-linearity as 2×10^{-5} sec^{-1}. Assuming this estimate to be (at each level) approximately three times the standard error of the instantaneous wind divergence and making use of horizontally uniform specific humidities of 7·2, 3·5, 1·3 and 0·2 g kg^{-1} at the 950, 750, 550 and 350 mb levels respectively we arrive at standard errors in average water-vapour divergence over the three months' period at these levels of approximately 1·3, 0·6, 0·2 and 0·0 g cm^{-2} $(100\ mb)^{-1}$ respectively. These errors are due to non-linear effects.

With regard to the errors in average water-vapour divergence due to errors in the computed average water-vapour transfers themselves, calculations show that the systematic errors in transfer given in section 4 lead to systematic errors in the average water-vapour divergence at the 950, 750, 550 and 350 mb levels which are negligibly small. In estimating the random errors in average water-vapour divergence at a given level the possibility of significant horizontal correlations between the random errors in zonal and meridional transfers at different stations must be considered.

Random instrumental humidity errors are uncorrelated in the horizontal, and calcu-tions show that the horizontal correlation coefficients between the lag errors in humidity the different stations are positive and of such magnitude that for a horizontal distance 300 km the value of 0·3 appears representative. Taking into account the fact that the nd errors are uncorrelated in the horizontal, calculations show that the horizontal rrelation between the random errors in both zonal and meridional transfers due to rors in the wind and humidity observations is negligibly small. Order of magnitude nsiderations also show that the horizontal correlation coefficients between the random mpling errors in both zonal and meridional transfers may be neglected for a horizontal paration of 300 km. From this discussion it follows that the horizontal correlation tween the total random errors in both zonal and meridional transfers is negligibly nall and will have no appreciable effect on the estimation of the errors in water-vapour vergence over the area under consideration.

Use of the standard vector errors in water-vapour transfer set out in section 4 leads standard errors in water-vapour divergence at the 950, 750, 550 and 350 mb levels nounting respectively to about 2·3, 1·9, 1·1 and 0·3 g cm^{-2} $(100\ mb)^{-1}$ for the three onths' period being considered. These errors are due to instrumental and lag errors humidity, wind errors, and sampling errors in transfer. They combine in mean square ith the random errors due to non-linear effects and give combined standard errors in ater-vapour divergence averaged over the three months' period amounting to 2·6, 2·0, 1 and 0·3 g cm^{-2} $(100\ mb)^{-1}$ respectively at the 950, 750, 550 and 350 mb levels.

A comparison of the corresponding values of water-vapour divergence given in able 2 shows that errors of this magnitude are not sufficient to invalidate individual lues of the computed divergence of water vapour or affect the main features of its stribution with height. In fact the main effect of the above errors is to introduce a ight degree of uncertainty into the estimated height (pressure) of the level of non-ivergence of water-vapour transfer. This uncertainty, however, has no relevance to ie main conclusions of this study.

For the vertically integrated average divergence of water vapour the systematic rors at each level (as already stated) are negligibly small and even when integrated ertically lead to a systematic error in the vertically integrated average divergence of ss than 0·1 g cm^{-2}. For the random errors in the vertically integrated average water-apour divergence it was found that, as with the errors in the vertically integrated average ansfers themselves, the correlation coefficients in the vertical between the divergence rors at different levels were not large enough to affect appreciably the estimation of ie error in the vertically integrated average divergence of water vapour over the area onsidered. As in the transfer calculations the error in the vertically integrated average ivergence was computed from the individual errors in divergence taken at the centres f layers 100 mb in vertical extent extending from the 1,000 to the 300 mb level. Errors or the layers intermediate to those computed above were obtained by interpolation. Use f these errors in divergence at the individual levels resulted in a standard error in the verage water-vapour divergence over the area for the three months' period considered f approximately 4 g cm^{-2}.

Compared with the computed total divergence of water vapour, amounting to g cm^{-2}, this standard error is very substantial and leads to the conclusion that although t each level the total divergence of water vapour for 100 mb layers can be determined vith sufficient accuracy, the total outflow of water-vapour integrated over all levels is ubject to considerable uncertainty. This uncertainty is in some measure due to the ircumstance that a comparatively large lower-level convergence of water vapour is very early balanced by a correspondingly large upper-level divergence thus leaving a relatively

small residual for the net outflow from the area concerned. It is also of some interest to note that over half the error is due to sampling uncertainties, and that these uncertainties can only be reduced by taking a longer period than three months or by increasing the number of observations per day.

7. Estimated precipitation and evaporation

Although the computed figure is subject to considerable uncertainty it has been estimated in previous sections that during the period there existed over the area an excess of precipitation over evapotranspiration, amounting to approximately 7 g cm^{-2}.

For a normal summer period such an excess appears to be improbable. Nevertheless it is well known (Glasspoole 1954) that the summer of 1954, at least as regards rainfall and sunshine, was far from normal. Table 3 has been compiled from data given in the *Monthly Weather Report* of the Meteorological Office, London, and shows the amount of precipitation and duration of sunshine experienced over the greater part of the area under consideration during June, July and August 1954, expressed as a percentage of the 1881-1915 average in the case of rainfall and of the 1921-1950 average in the case of sunshine.

TABLE 3. Precipitation and sunshine over southern England, June-August 1954 : percentage of normal

	June		July		August	
	Precipitation	Sunshine	Precipitation	Sunshine	Precipitation	Sunshine
E. England	126	71	125	72	189	74
SE. England	158	73	123	72	141	79
SW. England and Wales	170	75	114	74	107	78
Midland Counties	151	70	94	76	161	69

It appears from Table 3 that precipitation over the area was probably about 140 per cent of normal while sunshine probably averaged about 75 per cent of normal. The figures given by Glasspoole (1954) show that rather similar conditions prevailed over much of England and Wales. From these considerations it appears probable that a surplus of precipitation over evaporation might actually have existed over the area in question and it is of some interest to test this point further.

The total precipitation over the area for June, July and August 1954 was evaluated by measuring with a planimeter the areas enclosed between successive isohyets on the precipitation maps appearing in the *Monthly Weather Report*. In this calculation each area between successive isohyets was taken to have a uniform precipitation equal to the average of the precipitation values on the bounding isohyets.

The total amount of water vapour given off to the atmosphere by evapotranspiration from the earth's surface was calculated according to the methods set out in *Min. Agriculture and Fisheries, Technical Bulletin*, No. 4 (1954). For each month the calculations were carried out for 25 locations distributed as uniformly as possible over the area and their average was taken. The results of these calculations indicate that in June 1954 the average precipitation over the area was 3·8 in. while evapotranspiration amounted to 3·2 in. For July the corresponding figures were 3·3 in. and 3·2 in. respectively and for August 4·2 in. and 2·7 in. respectively. Thus for the summer of 1954 a total of 11·3 in., approximately

g cm^{-2}, of water was removed from the space above the area in question by way of ecipitation while only 9·1 in., approximately 23 g cm^{-2}, was added by evapotrans- ation. These figures give a surplus of approximately 6 g cm^{-2} of water for the period. is result is in good agreement with the figure of 7 g cm^{-2} obtained from considerations integrated water-vapour flux-divergence over the area concerned. However, considering accuracy with which the total precipitation over the area can be assessed, the some- at uncertain values for the evapotranspiration and the large standard error in the tically integrated water-vapour flux-divergence, such agreement must be regarded as some extent fortuitous. Nevertheless, in spite of the above uncertainties, the fact t such agreement can be obtained between results derived by entirely independent thods does tend to show that the vertically integrated water-vapour flux-divergence computed from the aerological data is probably of the right order and capable of being d in conjunction with other data to give a reasonably correct picture of the water balance a relatively small area.

8. Vertical transfer of water vapour

Over the period considered the regional water balance given in the previous section y be expressed as follows :

Water loss :	by precipitation (measured)	29 g cm^{-2}
	to storage (calculated)	1 g cm^{-2}
Water gain :	from flux divergence (calculated)	8 g cm^{-2}
	from evapotranspiration (required for balance)	22 g cm^{-2}

tually, empirical estimates of evapotranspiration gave 23 g cm^{-2} instead of the 22 g cm^{-2} en above but the latter is adopted here for internal consistency, the empirical value ing regarded as a check on the figure obtained from the aerological data and measured ecipitation.

From the data in Table 2 it may be deduced that approximately 18 g cm^{-2} are added tween the surface (1,005 mb) and the 950 mb level by convergence of water-vapour d from the above figures it follows that 22 g cm^{-2} are added by evapotranspiration. If n (as seems reasonable) no water is removed from this layer by precipitation, these ures imply an upward transfer of water vapour amounting to 40 g cm^{-2}. Similar con- lerations make it probable that, even after allowing (in a reasonable way) for the removal water at some levels by precipitation, the data in Table 2 imply an upward flux of ter vapour at each of the levels under consideration. It is naturally of some interest investigate the mechanism by which this upward transfer of water vapour is accom- shed and to attempt to assess the relative importance of large-scale and small-scale tical motions in the transfer process. To this end large-scale instantaneous vertical locities over the area were computed at 900, 800, 700 and 600 mb by the kinematic thod (Bellamy 1949, Sheppard 1949), and the large-scale vertical transfers of water pour evaluated by multiplication with instantaneous areally averaged specific humidities these levels. The material used here consisted of a random sample of 100 observations m the data under consideration. Vertical advective transfers were also computed for same sample by using temporal mean values for both the vertical velocity and the ally averaged specific humidity at each of the levels concerned. By this method the al and advective vertical (positive upwards) transfers of water vapour for the three nths' period were found to be respectively 15 and 16 g cm^{-2} at 900 mb, 12 and g cm^{-2} at 800 mb, 4 and 4 g cm^{-2} at 700 mb, $-$ 2 and $-$ 1 g cm^{-2} at 600 mb.

Although the above figures were obtained only for one particular sampling of th observational data, the extensive nature of the sample makes it probable that over th entire period large-scale eddy-transfer of water vapour contributed very little to th total vertical transfer. Thus at each level the vertical water-vapour transfer for the entir data may be approximately inferred simply from the temporal mean values of the vertic velocity and areally averaged specific humidity at the levels concerned. The results o such computations, applied to the entire data, are given in Table 4.

TABLE 4. VERTICAL FLUX (POSITIVE UPWARDS) OF AIR AND WATER VAPOUR, JUNE-AUGUST 1954

Level (mb)	950	850	750	650	550	450	350	
Areally averaged specific humidity	7·2	5·3	3·5	2·2	1·3	0·7	0·2	$\times 10^{-3}$ g g
Mass flux of air	+ 2·3	+ 4·3	+ 3·7	+ 2·7	+ 1·6	− 0·2	− 3·3	$\times 10^{3}$ g cm
Mass flux of water vapour	+ 17	+ 23	+ 13	+ 6	+ 2	0	− 1	g cm^{-2}

The errors involved in the computation of vertical velocities by the kinematic metho have been discussed by Sheppard (1949) and Weber (1952). From these studies it ma be inferred that the standard errors of the figures given in Table 4 for the vertical flu of water vapour are not likely to exceed 3, 2, 1 and 1 g cm^{-2} respectively at the 95 750, 550 and 350 mb levels.

Bearing in mind this uncertainty, it appears from Table 4 that the large-scale flu of water vapour is directed upwards at least up to the 550 mb level. Above this leve a weak downward flux is indicated but at these levels uncertainties in the computation make the reality of this downward flux somewhat questionable. It nevertheless seem clear that at 950 mb the computed large-scale upward flux of 17 g cm^{-2} is not nearl sufficient to provide the flux of 40 g cm^{-2} required by water-balance considerations. Abov this level the distribution of the vertical flux of water vapour required for balance cannc be determined with any precision owing to lack of knowledge of the amounts of wate removed from the various atmospheric layers by way of precipitation. It is readily seer however, that any reasonable distribution of this latter quantity with height leads to th conclusion that the large-scale flux is probably insufficient to satisfy balance requirement at all except the highest levels. At these levels both estimated and required fluxes ar small and the amount of agreement correspondingly uncertain.

It thus appears that although large-scale mean vertical motion is an important mech anism by which water vapour is supplied to the upper atmosphere over the area in question, the flux of water vapour carried by smaller-scale turbulent and convectiv systems is at least of equal importance in the total vertical transfer over the region durin the period under consideration. The important part played by small-scale eddy motion in the vertical turbulent transfer of water vapour thus stands in sharp contrast to th almost negligible role of eddy motions on the larger scale. This result might perhaps b regarded as unexpected, but recent work (Sheppard 1954) points to similar conclusion with regard to the vertical transfer of momentum.

9. CONCLUSIONS

The present investigation must be regarded as a preliminary attempt to obtai quantitative data on the horizontal transfer of water vapour in the atmosphere and t show how, at each level, the divergence of the horizontal water-vapour transfer field an

e associated transfer of water vapour in the vertical are related to the water balance a particular region. Owing to the fact that this work is limited to one particular period time and one particular region a fuller appreciation of the above processes must await proper extension of the methods used in this paper to other time-periods and geographical cations. This task calls for the routine evaluation of water-vapour transfers on a global matological basis and offers scope for international co-operation. Nevertheless, although nited to a particular period of time and geographical location, some results of interest ve been obtained and these are briefly summarized below.

(i) By an examination of the errors inherent in the basic aerological data and in e necessary computational procedures it is shown that, over a period of three months, o observations daily of wind and humidity, extending from the earth's surface to the 0 mb level and separated by 50 mb intervals at lower levels and 100 mb intervals at gher levels, are sufficient to give reasonably accurate estimates of the average water-pour transfer vectors at individual atmospheric levels and to provide satisfactory esti-ates of the average transfers integrated vertically throughout the atmosphere. It is und, however, that the greater part of the random transfer error is due to the sampling ocedure so that to realize fully the instrumental accuracy already present in the observa-ons more than two observations daily would be required.

(ii) It is further shown that by using observation stations at the corners of a adrangular area of side about 300 km and area about 9×10^4 $(km)^2$, transfer data of e precision obtained are sufficient to provide satisfactory estimates of the average total ater-vapour divergence at individual levels over the area for the period in question and obtain a reasonably accurate representation of the distribution of this quantity with ight. The average divergence of water vapour integrated throughout the atmosphere however, a relatively small quantity and it appears that this cannot be accurately termined from the aerological data employed here.

(iii) Average zonal total transfers of water vapour over southern England are rected eastward and reach a maximum of 400-500 g (cm 100 mb sec)$^{-1}$ below the 900 mb vel. Above this level these transfers decrease in an approximately linear manner with creasing pressure so that only about 1 per cent of the vertically integrated transfer curs above the 350 mb level. Average meridional total transfers are much smaller an those in the zonal direction, may be directed either northward or southward, and ow considerable differences in vertical distribution within a relatively small area. Vertic-ly integrated total transfers are of the same order of magnitude as those observed in titude 50°N over the coastal areas of North America (Benton and Estoque 1954).

(iv) Average zonal eddy-transfers are mainly directed eastward and account for ly a small percentage of the total zonal transfer. Average meridional eddy-transfers e mainly directed northward (irrespective of the direction of the total meridional flux) d form an important part of the total meridional transfer of water vapour.

(v) Although directed approximately from high to low values of specific humidity, erage eddy-transfer vectors at a given (geometric or pressure) level bear little quantitative lation to average horizontal or isobaric specific humidity gradients at the same level. fixing-length formalism is thus of little use in estimating large-scale turbulent transfer water vapour in the atmosphere.

(vi) The horizontal total water-vapour flux-divergence is distributed with height a characteristic manner. Convergence of water vapour takes place from the surface the 850 mb level while above this divergence of water vapour occurs, its magnitude radually diminishing towards zero at the highest levels. For the three months' period wer level convergence amounted to 31 g cm^{-2} while upper-level divergence amounted 23 g cm^{-2}. This distribution may show some features belonging to the general planetary

circulation but should most probably be regarded as mainly due to a regional circulatio between land and sea.

(vii) The average divergence of water vapour over the area, integrated throughou the atmosphere for the period in question, shows a net convergence of water vapou amounting to 8 g cm^{-2}. After allowing 1 g cm^{-2} for increase in storage in the atmospher over the area there is a net convergence of 7 g cm^{-2} to be accounted for by an excess o precipitation over evapotranspiration of approximately this amount. Although this figur is subject to great uncertainty it agrees well with the corresponding value of 6 g cm^{-2} deduced from measured rainfall and empirical estimates of evapotranspiration.

(viii) Over the period in question the computed distribution of the total averag water-vapour flux divergence almost certainly implies an upward flux of water vapou over the area at each of the levels concerned. Direct computations of the large-scal flux of water vapour shows that this flux is indeed directed upward, at least up to th 550 mb level, but that its magnitude is not nearly sufficient to provide the flux require by balance considerations. It is inferred that a considerable part of the required upwar flux of water vapour is accomplished by turbulent and convective systems of essentiall smaller scales.

(ix) Large-scale eddy-motions in the vertical direction associated with passing cyclones and anticyclones and their associated upper-level troughs and ridges are show to be ineffective in accomplishing large-scale transfer of water vapour in the vertica Almost all of the large-scale vertical water-vapour transfer occurring over the area durin the period in question was associated with large-scale mean vertical motion.

Acknowledgments

The author wishes to express his thanks to Dr. P. Whittle of the Applied Mathematic Laboratory, New Zealand Department of Scientific and Industrial Research, for advice on statistical questions, to Miss S. Fraser for assistance with the computations and to the Director of the New Zealand Meteorological Service for permission to publish.

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preliminary study of atmospheric waves using radiosonde data

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Summary

Evidence of atmospheric gravity waves, almost certainly due to orography, is obtained from a study British radiosonde data. It is shown that such data can usefully be exploited to study some aspects of flow of air in the neighbourhood of mountains.

1. Introduction

Many observations of mountain airflow effects in general, and of lee waves in rticular, have come from the pilots of gliders and aircraft. Although such observations ve contributed greatly to our understanding of the problems of mountain airflow, their plication is nevertheless limited. Furthermore, field investigation of the problem using ders or aircraft presents formidable difficulties, apart from being extremely expensive. is natural to enquire, therefore, to what extent the data from routine radiosonde ascents n be used to study atmospheric waves. This paper is concerned with a trial determination d analysis of the vertical air currents revealed by six months' radiosonde data from four itish upper-air stations.

2. The data and their treatment

The data used were from the upper-air stations at Leuchars, Stornoway, Aldergrove d Liverpool for the six-months period Nov. 1953 to April 1954. Both the 0200 and 00 GMT soundings were used, making a total of about 1,440 soundings in all. The positions the stations, together with the main features of the topography, are indicated in Fig. 1.

The so-called Väisalä diagram which is plotted at British radiosonde stations includes lot of pressure against time during the ascent. If the rate of ascent of the balloon varies bstantially and systematically with height, it is easy to recognize this visually from the essure-time plot, and an examination of these curves was found to be adequate for the rpose of selecting the cases worth studying in greater detail. In this way, 61 occasions re selected when the rate of ascent of the balloon varied by approximately $\pm$ 300 ft/min more; consideration of the errors of estimating vertical currents from radiosonde data dicated that it would not be profitable to study evidence of smaller vertical currents.

Readings of the audio-frequency of the note controlled by the pressure element of the nde are plotted against time by the operator at intervals of about $\frac{1}{2}$ min, the exact erval depending on the rate of revolution of the sonde's ' windmill.' From these curves, e mean rate of change of this frequency, dc/dt, in cycles sec^{-1} min^{-1}, was evaluated over ch minute of each sounding, up to 30 min, using a protractor and a conversion table. In nilar fashion, the corresponding rates of change of pressure with the frequency, dp/dc, re evaluated from the calibration curves. Multiplication of these two quantities gave e mean rate at which the balloon crossed the isobaric surfaces, dp/dt, over each minute. lues of dp/dt were converted to the vertical velocities, dh/dt, by multiplying by the ues of dh/dp appropriate to the mean pressure and temperature over each minute.

Figure 1. Location of upper-air stations used in the analysis and main features of the topography.

The above procedure for the evaluation of the rates of ascent is one of several alternative methods. It was adopted as the best compromise between convenience and accuracy. As would be expected, as the radiosonde is not designed to measure vertical air currents, the currents which can be detected are limited by errors of technique and measurement. An assessment of the errors was made, from which it was concluded that up to the 30th minute of a sounding vertical air currents exceeding about 120 ft/min could be detected (at higher levels the errors of the procedure would increase rapidly).

An attempt was made to seek independent confirmation of the results from the radar data, which comprise readings of range, elevation and azimuth every minute. The errors in vertical air currents deduced from these data were also estimated. The errors depend greatly on range and to a lesser extent on elevation, and although reasonably accurate

rtical velocities can be determined in the first few thousand feet, the errors rapidly rease upwards (e.g., about 600 ft/min at a range of 30,000 yd).

Graphs of the rate of ascent against height of balloon were plotted and a mean rate ascent was inserted by eye in each case. The graphs were then re-plotted in terms of partures from the mean rate of ascent, i.e., vertical air currents. Most of the cases owed evidence, often striking, of vertical currents which were periodic along the path the balloon and in some cases of substantial magnitude. These were strongly suggestive gravity waves, presumably of orographic origin, but it must be realized that other tors could give rise to such variations, for example : convective cloud, intermittent ng, or precipitation. Cases when factors of this sort might conceivably have caused e observed variations in the rate of ascent were rejected on the basis of a study of the noptic charts. There remained after this sifting process 48 cases in which the deduced rtical air motions could be regarded with considerable confidence as being manifestations gravity waves.

3. Examples of wave motions and general comments

In Figs. 2 to 7 we give some examples of the graphs of vertical velocity of the air otted against height of balloon. Each graph comprises two curves, the thick curve ing that deduced from the pressure observations of the radiosonde and the thin curve at from the radar data. The areas between the lines AB, CD for the pressure curves d between OX, OY for the radar curves, define the zones in which individual values

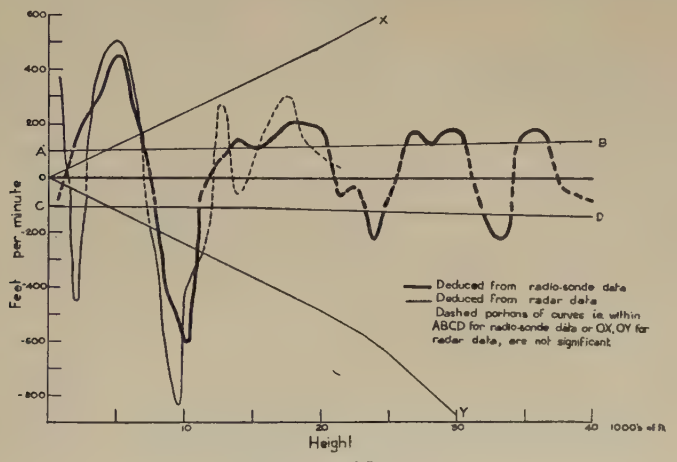


Figure 2. 4 Nov. 1953.

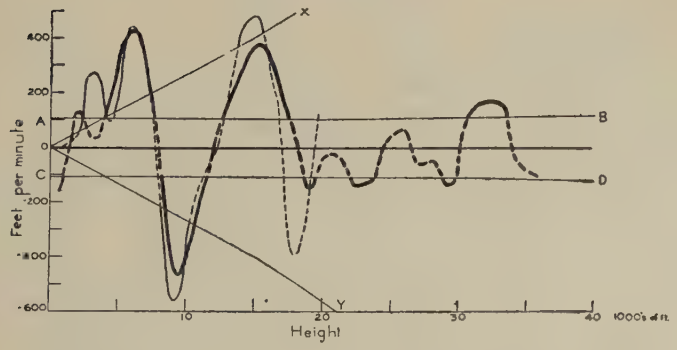


Figure 3. 21 Dec. 1953.

Figures 2 and 3. Vertical air currents – Leuchars, 1400 GMT.

of vertical velocity are not significant (5 per cent level), on the basis of the assessment of errors carried out. Within these zones the curves are plotted in dashed form. As already indicated, the radar data are more sensitive to vertical currents in about the first five thousand feet, but above this level the pressure data are much more reliable for the purpose in hand. Where a number of successive values are consistently of the same sign they may be significant even if within the 'zone of no significance' because the latter zone applies strictly to individual values. In Fig. 4, the radar data were derived from a separate sounding launched an hour or so later than that from which the pressure data were obtained.

The examples, which are typical of the 48 occasions found, show clear evidence of vertical currents which, as measured by the balloon during its oblique ascent, were periodic in the vertical. Much of the curves obtained from the pressure data lie outside the zone of no significance and even where they lie within the zone there is in general a good measure of consistency with the remainder of the pattern. Furthermore, the corresponding curves from radar data confirm to a convincing degree the pattern of vertical motion in the lower troposphere, at least as regards the general form if not always the magnitude. It may also be mentioned that on about half the 48 occasions there was confirmation in the form of aircraft reports of waves (viz. reports collected at Northolt Airport for the same period and used by Pilsbury (1955) in an analysis of aircraft reports of waves). There is, therefore, no reason to doubt the reality of the waves deduced here from radiosonde data.

In the majority of cases, the amplitude of the vertical velocity perturbation attains a pronounced maximum in the lower troposphere (usually from 5,000-10,000 ft above the ground) and decreases sharply above. Sometimes, however, the amplitude is maintained to considerable heights, as for example in Fig. 5. In no case was there convincing evidence of the amplitude increasing with height in the upper troposphere.

In some cases waves were present on two or more successive ascents, and also at more than one upper-air station, showing that at least sometimes waves have a fair degree of persistence in space and time. It is interesting to note, too, the agreement on Fig. 4 between the radar and pressure data which were not in this case simultaneous. However, this agreement must be to some extent fortuitous since small temporal changes in wavelength would change the location of the crests and troughs relative to the point of release of the balloon.

The case which gave rise to the largest amplitude waves (with vertical velocities of about 1,000 ft/min) is included in these examples, viz. Fig. 6, referring to 15 April 1954. For two or three days about this time waves were widespread and were the subject of many aircraft reports, one of which refers to vertical currents of 1,900 ft/min in the

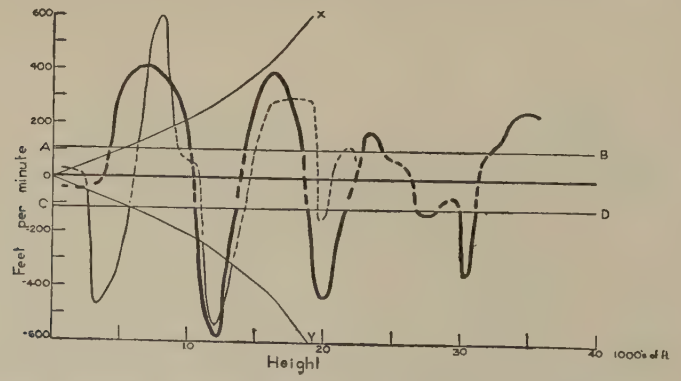


Figure 4. Vertical air currents – Leuchars, 1445 GMT, 3 Jan. 1954 (The radar data were obtained from a separate sounding an hour or so later).

egion of Edinburgh (see Pilsbury 1955). In addition, it may be noted that the violent urbulence encountered by a Canberra at 40,000 ft near Edinburgh and described by D. C. E. Jones (1955) occurred on the evening of 14 April 1954. Strong waves were evident from the Leuchars sounding nearest in time (1400 GMT), 14 April 1954, although not as large in amplitude as those on Fig. 6.

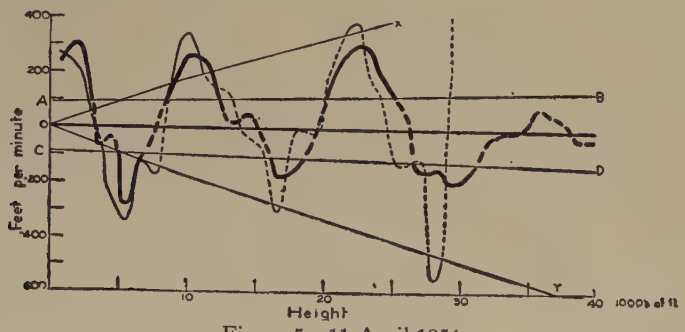


Figure 5. 11 April 1954.

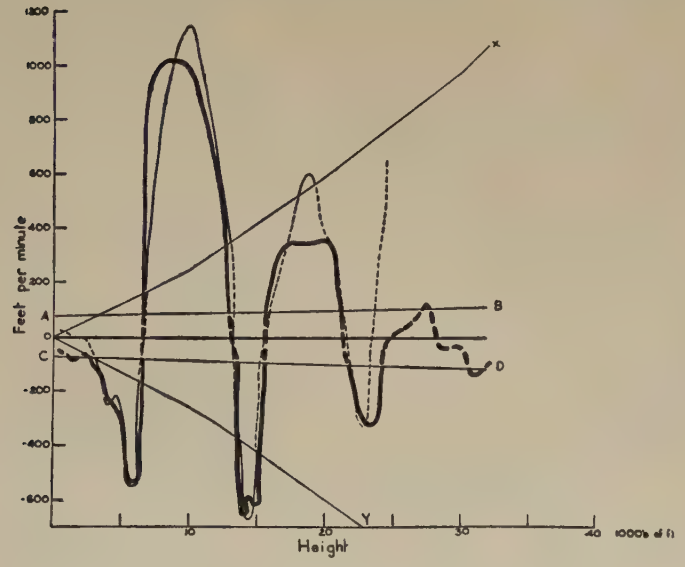
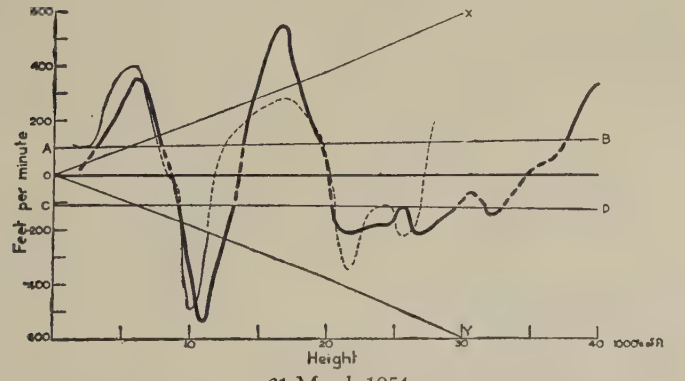


Figure 6. 15 April 1954.

Figures 5 and 6. Vertical air currents – Leuchars 1400 GMT.



21 March 1954.

Figure 7. Vertical air currents – Liverpool, 1400 GMT.

4. Some statistical results

(a) *Distribution amongst stations*

The 48 cases were divided amongst the stations as follows :

Station	No. of cases
Leuchars	27
Liverpool	14
Stornoway	4
Aldergrove	3

We should expect the frequency of detectable waves to be closely dependent on the character of the topography upwind of the stations in the direction of the prevailing wind and this no doubt accounts for the wide variation in the frequency of waves at the four stations.

(b) *Distribution of maximum vertical currents*

The frequency distribution of the maximum vertical currents, based on the pressure-frequency data, is illustrated by the histogram at Fig. 8. As expected, the distribution is J-shaped, indicating increasing rarity the larger the vertical current.

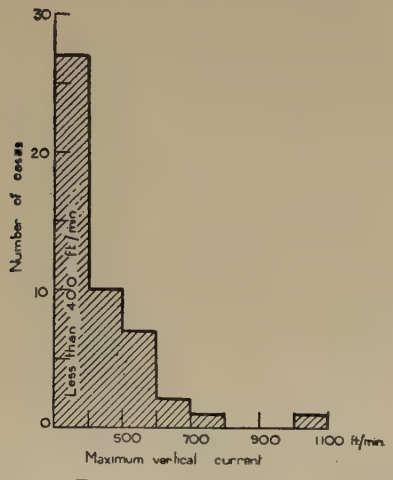


Figure 8. Frequency distribution of maximum vertical currents.

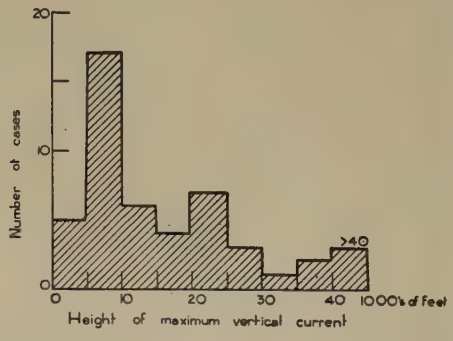


Figure 9. Frequency distribution of heights of maximum vertical currents.

The distribution of the heights at which the balloon encountered the maximum vertical current is given in Fig. 9. The main feature is the pronounced maximum in the layer 5,000 to 10,000 ft. Generally speaking, the theory of mountain airflow leads us to expect the amplitude of lee waves to increase upwards to some level in the lower troposphere and to decrease at higher levels. This picture is supported by the present data. It is not thought that the minor secondary maximum from 20,000 to 25,000 ft is significant.

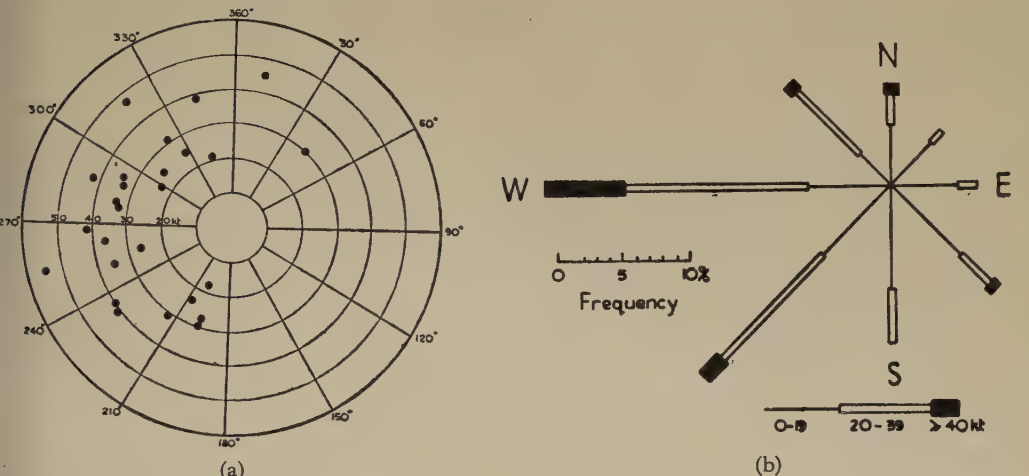
(c) *Seasonal and synoptic variations*

The occasions of waves were distributed amongst the six months as follows :

Month	No. of cases
November 1953	10
December 1953	2
January 1954	14
February 1954	4
March 1954	10
April 1954	8

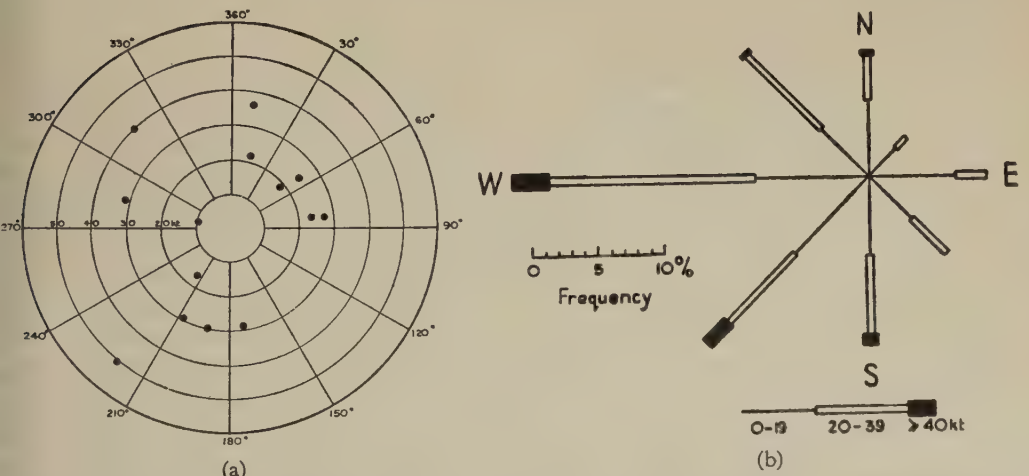
We know that the winter produces more situations favourable for waves than other times of the year because of the greater frequency in winter of strong winds and high static stability in the lower troposphere. However, the only significance to be attached in the above table to the low frequency of waves in Dec. 1953 and Feb. 1954 probably arises from the variability of the British climate. These particular two months were on the whole much quieter and less windy than is usual over Britain.

The theoretical condition for lee waves deduced by Scorer (1949) is not one which can be applied with precision in practice and it is well known that the condition can be satisfied in a wide variety of synoptic situations. However, in a high proportion of the present cases, viz. 30 out of 48, the radiosonde station was near a front – sometimes ahead of a warm front or near an occlusion, but more commonly to the rear of a cold front. Such a location generally provides the stability in the lower troposphere and the increasing winds with height, both of which tend to favour waves. Furthermore, it was clear from the time sequence of the occasions of waves at the different upper-air stations that the



(a) (b)

Figure 10. (a) 900-mb wind speeds and directions at Leuchars for cases of waves. (b) 900-mb wind-rose for Leuchars for all ascents Nov. 1953-April 1954.



(a) (b)

Figure 11. (a) 900-mb wind speeds and directions at Liverpool for cases of waves. (b) 900-mb wind-rose for Liverpool for all ascents Nov. 1953-April 1954.

region relative to the front where waves occurred tended to be advected with the front and to be maintained with it during the passage of the front across the British Isles. The remaining cases mostly occurred in warm sectors or anticyclonic westerlies, both of which are characterized by low-level stability and increasing winds with height.

(*d*) *Wind speed and direction*

In Figs. 10 (a) and 11 (a) are given in the form of polar diagrams the 900-mb wind speeds and directions at Leuchars and Liverpool for the wave cases. The mean 900-mb wind speed was 32·1 kt at Leuchars and 28·4 kt at Liverpool; these appear to be a little above normal for the winter months, although not greatly so.

As regards wind direction, it can be seen that all the Leuchars cases except two occurred with winds between N and S through W (i.e., directions with high ground upwind). In one case which might appear suspicious, viz., that with a NE wind, the wind below 900 mb was well backed and was sharply anticyclonic with Westerlies in extreme N. Scotland; the low-level flow had, thus, a predominantly land track. The largest-amplitude waves were concentrated in the NW'ly quadrant where the highest mountains, upwind from Leuchars, exist. A 900-mb wind-rose for Leuchars, covering all occasions during the six months period, is given at Fig. 10 (b).

The wind directions at Liverpool, Fig. 11 (a), were much more varied, as might be expected in view of the local topography. We will refrain from speculating as to whether waves with NW'ly winds owed their origin to the effect of the Isle of Man or to the hills of Northern Ireland, but will remark that lenticular clouds have been observed from the Lancashire coast stretching across the Irish Sea. A wind-rose for Liverpool, covering the six months period, is given at Fig. 11 (b).

Of the 48 cases as a whole, the change in wind direction from 900 to 500 mb was less than 20° in 36 cases and in only 2 did it exceed 30°. This confirms the well-established conclusion that relative constancy of wind direction with height is required for well-developed lee waves.

The four cases at Stornoway all occurred with strong SSW'ly winds; the airflow would thus have passed over the high ground in the SSW of the Island of Lewis itself. The Aldergrove examples were all with NW'ly winds.

(*e*) *The wavelength*

In many of the graphs of vertical motion against height of balloon, of which Figs. 2-7 are examples, a dominant wavelength in the horizontal stands out unmistakably. In fact, it was found possible in 26 cases to obtain a good estimate of the dominant horizontal wavelength, as the average of about 5 or 6 assessments over different portions of the curve, assuming vertical wave fronts. The average horizontal wavelength for the 26 cases was 6·7 mi (standard deviation 3 mi), the extreme values being 2·6 mi and 14·4 mi. These values are consistent with observational evidence and of the order expected from theoretical considerations.

Perturbation theory enables us to set down equations which formally determine the horizontal wavelength, but a general analytical solution is not possible owing to the complex way in which variations of static stability and wind in the vertical are involved. However, it seems clear that generally speaking the wavelength should increase with the mean wind speed through the troposphere. This conclusion is strikingly borne out by Fig. 12 which is a plot of the wavelength in the 26 cases against the corresponding wind speed meaned over the layers contributing to the wavelength assessments. The

correlation coefficient is 0·91, which appears sufficiently high to justify estimates of wavelength from mean tropospheric wind speed alone.

In Fig. 13 is plotted the maximum vertical component of wind in the Leuchars examples against the wavelength, when an estimate of wavelength was possible. Clearly there is no linear relation; indeed there are signs of a peak maximum vertical velocity for a wavelength of about 8 mi with smaller vertical velocities for both shorter and longer wavelengths, and a rough curve has been drawn by eye expressing this trend. This appears to offer some support for the conclusion reached by Corby and Wallington (1955), viz., that a kind of resonance effect would be expected to occur when the natural wavelength of the airstream matched the dominant wavelength component of the topography. Admittedly the points on Fig. 13 represent a variety of wind directions and therefore the support is by no means conclusive; sufficient data to enable such graphs to be plotted for distinct wind directions should throw more light on this aspect.

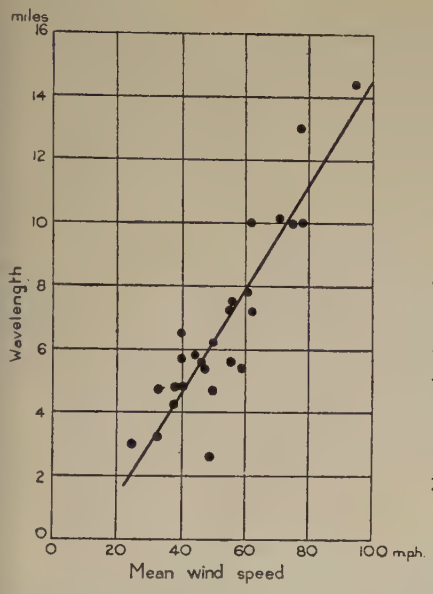


Figure 12. Relation between horizontal wavelength and mean wind speed (correlation coefficient 0·91).

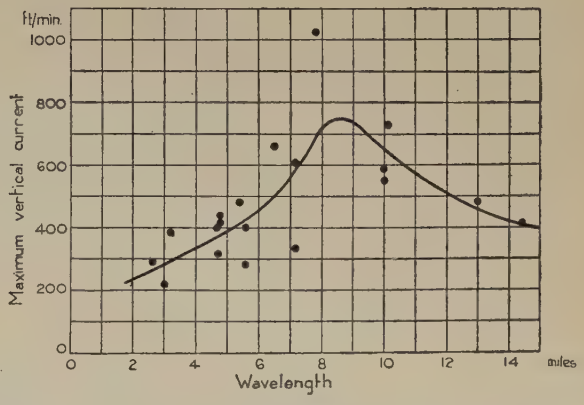


Figure 13. Relation between maximum vertical current and wavelength for Leuchars cases.

5. Apparent distortion of temperature structure

It was pointed out by Scorer (1953), that if a radiosonde balloon passes through waves, the temperature structure measured by the sonde will differ from that of the undisturbed stream and, indeed, that spurious inversions might appear on this account. If static stability is expressed, in the usual way, as $\theta^{-1}\,\partial\theta/\partial z$ where θ is potential temperature, and we neglect perturbations of horizontal velocity, then it may readily be shown that the apparent stability will be $W/(W+w)$ times the true stability, where W is the rate of ascent of the balloon relative to the air and w is the vertical component of wind. Thus in an ascending current stability will appear to be reduced and vice versa. In the extreme case of a descending current which brought the balloon to rest (in the vertical), the sonde would indicate an ideally sharp inversion whatever the undisturbed lapse rate.

The above remarks prompt us to enquire to what extent the temperature structure measured by a radiosonde passing through waves is in fact distorted in such ways.

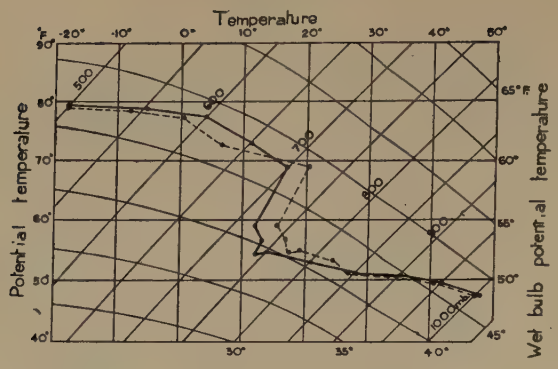


Figure 14. Leuchars, 1400 GMT, 4 Nov. 1954.

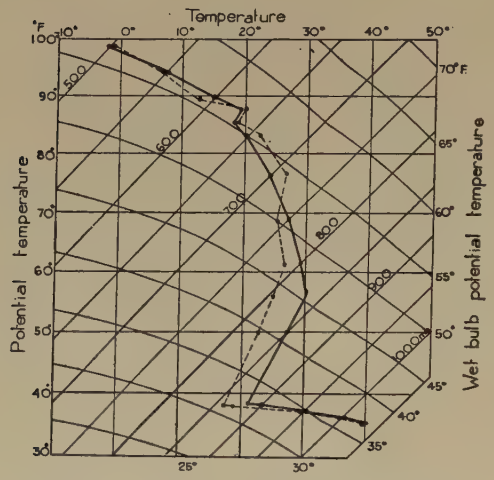


Figure 15. Leuchars, 0200 GMT, 3 Jan. 1954.

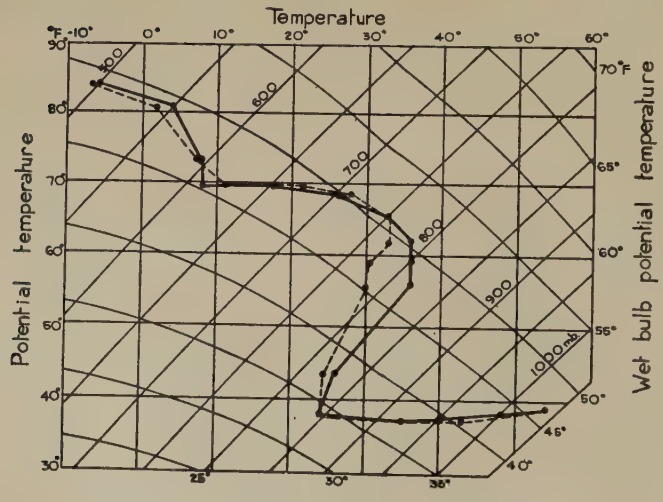


Figure 16. Leuchars, 1400 GMT, 15 April 1954.

Figures 14, 15 and 16. Apparent distortion of temperature structure by waves. Full curve – temperatures as reported; dashed curve – corrected version.

If we represent the displacement of a streamline from its undisturbed level by

$$\zeta = a \sin\left(\frac{2\pi}{L}x + \epsilon\right) \quad . \quad . \quad . \quad . \quad (1)$$

where a = semi-amplitude, L = wavelength and ϵ is a phase angle. Then

$$\frac{\partial\zeta}{\partial x} = \frac{w}{U} = \frac{2\pi}{L} a \cos\left(\frac{2\pi}{L}x + \epsilon\right) \quad . \quad . \quad . \quad (2)$$

and

$$[w]_{\max} = \frac{2\pi U a}{L} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where U is the horizontal component of wind.

In a number of cases an estimate of the wavelength, L, was made, values of $[w]_{\max}$ were read from curves such as Figs. 2 to 7, and thus from Eq. (3) the semi-amplitude, a, was obtained at several levels. The value of a at other levels was obtained by interpolation and the displacement at any level deduced from Eqs. (1) and (2). This provides a basis for correcting the temperature structure, subject to the important assumption that the centre line of the wave form at any level is the same as the corresponding undisturbed level upstream. A number of tephigrams were corrected in this way and three examples are given in Figs. 14 to 16. In these diagrams the temperature structure as measured by the sonde is plotted as a full line and the corrected version is left dashed. Layers of steep lapse rate are not, of course, much affected but layers of marked stability and in particular inversions may be modified considerably. If a layer containing an inversion in the undisturbed stream is subjected to a maximum vertical current in the wave system as the balloon passes through it, the intensity of the inversion will be modified; if the balloon passes through such a layer in the crest of a wave there will be little effect on intensity but the temperature level will appear to be lower, whilst if measured in a wave trough the temperature will be higher. There is evidence of all these effects in the examples of Figs. 14-16, but it is emphasized that the precise effect in any given case must be almost fortuitous : release of the balloon a mile or two up- or down-stream would change the measured temperature structure.

In no case examined in this way did it appear that any of the substantial inversions, which are commonly found when lee waves occur, were completely spurious. The main effect seems to be to change the intensity or level of strong inversions. The dew-point curves are modified in a similar fashion.

Conclusions

Where upper-air stations are to the lee of hills or mountains, the routine radiosonde data can provide a valuable means of studying orographic lee waves, and of building up background statistics of such phenomena. In the present preliminary study, support was found for several conclusions which have been deduced from perturbation theory. The results obtained here suggest that profitable work might be done by using a mobile radiosonde station amongst mountains. Furthermore, useful indications of the occurrence of waves could be given from existing radiosonde stations on a current basis; at present little or no observational data on waves reaches aviation forecasters, and radiosondes could be exploited to help to fill this gap.

Acknowledgments

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An inherent precipitation mechanism in cumulus clouds

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Summary

In a fair-weather cumulus, the largest droplets, which are of 18 microns radius, grow too slowly by differential settling to become precipitation (although the time to reach raindrop size from 50 μ, for instance, would be comparatively short). Even at 18 μ their terminal speed is sufficient for growth, were it not that most of the droplets they encounter are small enough to avoid capture. Using Langmuir's collision efficiencies, the calculated time to grow from 18 to 50 μ radius is 2 hr. This time would be reduced to 12 min if the cloud sample had first ascended until there were 4 g of condensed water per kg of air, as must take place in the updraught of a cumulus congestus.

Condensation and coalescence actually occur simultaneously and the first 50 μ droplets appear at about the 6 g/kg level; this is about the level at which the first radar echo should, and does, appear. The apparent descent of the echo can be accounted for by droplet growth at lower levels. The rapid development of precipitation in cumulus can be explained thus without invoking giant nuclei or ice.

The intensity required for turbulence to be a potent coalescing agency was found to be greater by a factor ten than the intensity of turbulence deduced from aircraft measurements.

1. Introduction

It is well established that moderate to heavy rain often falls from ' warm ' convective clouds, where the Bergeron mechanism is not applicable. A major problem is the growth of droplets from a radius of about 10 μ, which has been attained by condensation on to hygroscopic nuclei (Howell 1949 and others) to a broad range of radii extending to beyond 30 μ; after that their fall velocity is sufficient to ensure rapid growth by collision with smaller drops.

East and Marshall (1954) made a survey of droplet-size distributions measured in cumulus by Diem (1942, 1948) and by Weickmann and aufm Kampe.* It was found that fair-weather cumulus clouds have distributions (resembling that of the curve $t = 0$ in Fig. 2) which are lacking in droplets of radii greater than 18 μ; a feature recently confirmed by Battan and Reitan (1956). Heavy cumulus (cumulus congestus and cumulonimbus) distributions, on the other hand, extend to 100 μ radius and beyond, and the major part of their liquid-water content lies above 40 μ radius. The cloud air which contains these droplets must have risen through the condensation level originally and acquired a fair-weather type of spectrum first. Zaitsev (1950) found a progressive change of spectrum with increasing altitude inside one cloud, from a fair-weather type spectrum towards the heavy type of distribution.

Heavy cumulus distributions have typically about four times the liquid-water content of fair-weather cumulus, which can only be accounted for at this stage in the life-history of the cloud by the condensing of extra liquid water on to the droplets. Measurements also show that heavy cumulus has several times fewer drops per cm^3 than fair-weather cumulus. Though some spreading-out must occur through the increase in specific volume of the air with altitude, the increase would not usually exceed a 1·5 to 1 change, leaving a large change in population which must be due to coalescence between droplets. It will be shown that the combination of condensation and coalescence can readily account for the observed change of the spectrum and that it provides a likely mechanism for rain formation.

* Private communication.

In East and Marshall (1954) coalescence (induced by gravity and by turbulence) was considered mainly through collision probabilities, though some growth curves were calculated for gravity alone. Other writers have calculated the growth of a droplet in static circumstances, with all the other droplets remaining constant in size and number. The present paper gives the results of calculations of the effect of various factors on all the droplets at once. This step in the direction of reality required such extensive computation that other, smaller, idealizations had to be made. As a result, the calculated speeds of development do not have high absolute accuracy, but indicate very satisfactorily the relative importance of the various factors.

It was shown in the earlier paper that the collision probabilities for gravity alone in a fair-weather type distribution were too small for the required development but that sufficiently intense turbulence could bring about rapid changes. Later calculations have shown within fairly narrow limits what minimum intensity would be required to bring about the rapid development observed, and a review of the experimental data (not intended for, and not altogether suitable for this purpose) has found an intensity about ten times too low (East 1957).

In § 3 it is shown that sufficient cloud density (available in a sufficiently deep cloud) is all that is necessary to make coalescence a potent mechanism; it can then account for the observed drop-size distribution, the observed height and behaviour of the initial radar echo, and above all, for the time-scale of these events.

2. Coalescence

(a) *Growth by collision*

A drop, radius s, mass m_s, grows by coalescence with droplets whose radii r lie between 0 and r_1 at the rate

$$\frac{dm_s}{dt} = \int_0^{r_1} \pi (s + r)^2 \, UE \, w(r) \, dr \qquad (1)$$

where $w(r)\,dr$ = liquid water content of droplets of radii r to $r + dr$, $U = v_s - v_r$, the relative fall velocity of drop and droplet, and E = collision efficiency. Fall velocities were computed using Stokes's Law; above 18 μ radius a correction factor depending on Reynolds's number, derived from Schiller's (1932) survey of experimental results, was used. (The temperature and pressure vary throughout the cloud, but in this section air is taken to have constant density and viscosity $\rho_a = 0{\cdot}96 \times 10^{-3}$ g cm^{-3} and $\eta = 1{\cdot}72 \times 10^{-4}$ g cm^{-1} sec^{-1}. These are the values for 0°C and 760 mb, roughly the conditions in which precipitation first forms in temperate latitudes.) It can be shown that Cunningham's (1910) correction to fall velocity and Brownian movement have negligible effect on collisions between cloud droplets.

It was assumed as a basis for calculation that Langmuir's (1948) equation for collision efficiency applies to small droplets of comparable size (Fig. 1, curve E); the experimental evidence, some of which conflicts with the theory, is not yet comprehensive enough to replace it. (Gunn and Hitschfeld 1951, Adderley 1953, Sartor 1954, Telford, Thorndike and Bowen 1955, Schotland 1956, Kinzer and Cobb 1956). The use of Langmuir's equation is discussed further in Appendix 1.

(b) *Development of drop-size distributions*

The limit of the integral, r_1, would be simply s if the largest drops only were allowed to grow. In this section, however, the simultaneous growth of a range of drop sizes s was calculated. An arbitrary value of 13 μ was given to r_1, and s ranged from r_1 upwards, so

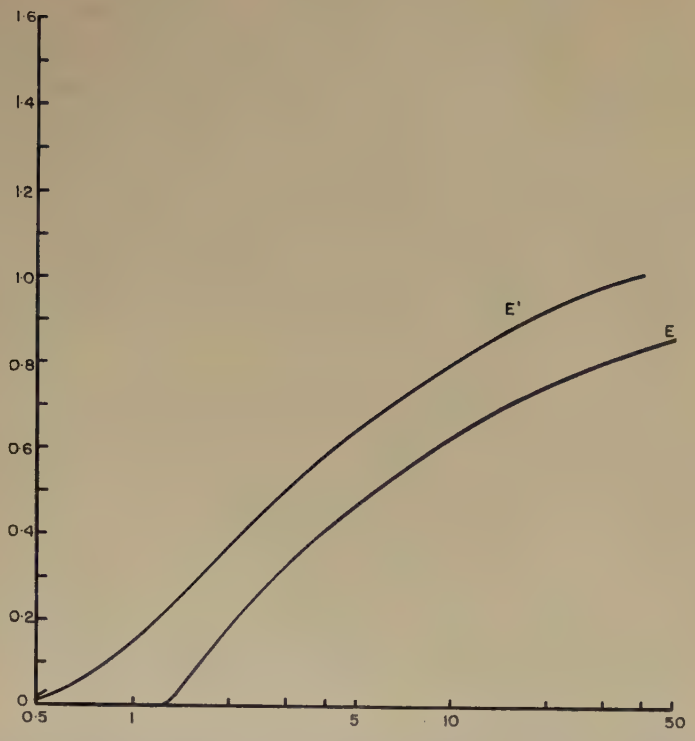


Figure 1. Curve E. Collision efficiency versus Langmuir's parameter K (Eq. (6), appendix 1). Curve E'. Invented collision-efficiency curve on which revised calculations of droplet-size development were based.

that the only collisions considered were those in which a 'drop,' radius $s > r_1$, would grow, while a 'droplet,' radius $r < r_1$, would disappear. This artificial limitation greatly simplified the computations, by elimininating the possibility that a particular droplet might grow for a while by sweeping up smaller droplets and then itself be swept up by a larger droplet, but it also makes the calculated growth rates slower so that the development times err on the side of being too long. The starting point was an idealized fair-weather distribution with droplet radii running from 3 to 18 μ, having the shape of Fig. 3 (a) of East and Marshall (1954), but with water content 1 g/m^3. (It appears as curve $t = 0$ in Fig. 2.) For purposes of calculations, the droplets below 13 μ radius were grouped into 5 classes, and so were the drops between 13 and 18 μ. The quantity $\pi (s + r)^2 UE$ was plotted against s for values $r = 4$, 6, 8, 10, 12 μ. The simultaneous growth of the five classes of drop s at the expense of the droplets r could then be computed, and in this way the change of drop-size distribution with time was calculated step-by-step, assuming the total liquid water content to remain at 1 g/m^3 (25 collision rates being calculated at each step). The finite time interval for each step also tends to give slow growth rates. In Fig. 2 the depletion of the smaller droplets and extension of the curve to larger radii produce a distinct development toward the heavy cumulus type of distributions, but take a time (nearly two hours) which cannot be realistically considered in the lifetime of a parcel of air in cumulus.

Although the low collision efficiencies assumed had a good deal to do with the slowness of the development, the result is not as critically dependent on the particular form of Langmuir's equation as one might expect. Suppose, for example, the collision efficiency were as shown by E' instead of by E in Fig. 1. It would mean having efficiencies ranging up to 20 per cent where Langmuir's equation gives zero, and at least 15 per cent above

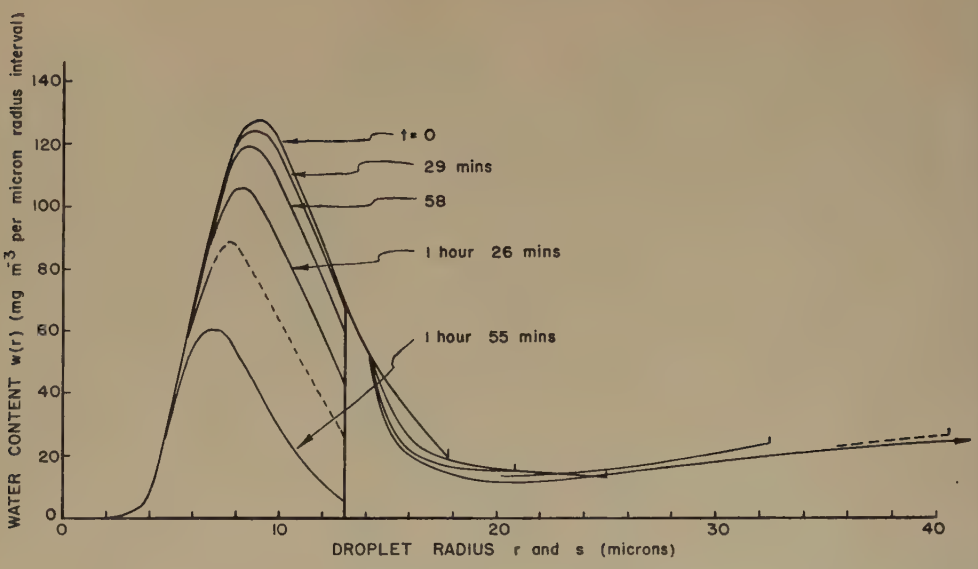


Figure 2. Modification of water-content distribution by coalescence. The curve $t = 0$ is for idealized fair-weather cloud (East and Marshall 1954, Fig. 4 (a)). Drops larger than 13 μ are allowed to sweep up droplets smaller than 13 μ : the curves for later times show the result. (Between 14 and 20 μ several of the curves almost coincide. In this region only the curves for $t = 0$, 29 and 58 min and 1 hr 55 min are shown). After 1 hr 55 min (last curve) the largest drops reach 49 μ.

Langmuir's values elsewhere. This is a more drastic revision even than some recent experiments suggest. The development which would result from the use of E' instead of E was calculated (for quite another purpose, as it happens). The production of a heavy cumulus type of distribution was somewhat more rapid, but even after one hour the largest droplets had only reached 40 μ. It does not seem likely that any foreseeable revision of the collision efficiency equation will alter the conclusion that gravity alone is not an important precipitation mechanism. The observed stability of fair-weather cumulus tends to confirm this view.

It was suggested by Cochet (1952) that electric charge might influence coalescence; he considered a cloud of neutral droplets of radius 8 μ, with water content 2 g/m^3, and showed that a droplet of radius 15 μ carrying a charge of 4×10^{-4} e.s.u. would grow very rapidly by coalescence. This charge, however, is equivalent to about 800,000 electrons; it seems much too large to be found in a warm cumulus in the early stages of precipitation. In a private communication, Dr. Ross Gunn reports that in a cloud in neutral air the mean equilibrium charge on each droplet would approximate 19 elementary units, at a radius of 15 μ. Considering two such droplets differing sufficiently in radius (and therefore having sufficient relative velocity) that their collision efficiency would be only just zero when uncharged, approximate calculations show that equal and opposite charges of 19 units would increase the collision efficiency to 0·005 per cent at the very most, and probably to very much less than this.

So far, following well-established precedent, we have treated growth as a continuous process. Telford (1955) points out, however, that drops grow discontinuously, increasing by a finite amount at each collision. He has set up a statistical theory of this effect, and applied it to some aspects of cloud coalescence. In the early stages of growth, while the growing drops are not much bigger than the droplets, the effect speeds up the growth of a lucky few and broadens the drop distribution considerably, but subsequently the growth rates approach the ' continuous ' values. Referring to Fig. 2, the discontinuous

process might shorten the 58 min required to extend the curve from 18 to 25 μ, but would not greatly reduce the second hour which is occupied in going to 49 μ. It is difficult to apply Telford's results directly to ours because he assumed a collision efficiency of 100 per cent throughout.

Mason (1954) describes experiments in which glass spheres of equal size were brought into contact by a shearing action in the liquid in which they were suspended. At concentrations of a few per cent a sphere spent much of its time in contact with other spheres. At the concentration of a few parts per million by volume which is found in clouds, however, collisions between water droplets from this cause would be very rare even when the shear is of the order 10^2 sec^{-1} (estimated value for the interior of a vigorous cumulus cloud).

3. Condensation and its effect on coalescence

Nearly all collisions in a cloud occur between a larger and a smaller droplet. Much has been said and written about the effect of condensation on the larger droplet, particularly where giant nuclei are involved. The effect on the smaller droplet, however, appears to have been overlooked. To investigate this, we consider first the law of condensation on droplets.

(*a*) *Condensation*

As cloud-filled air ascends, it tends to become more supersaturated but vapour diffuses on to the droplets where it is removed by condensation. The rate of increase of mass of a droplet is proportional to the supersaturation S and to its radius r, thus

$$\frac{dm_r}{dt} = F S r \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This equation neglects the influence of curvature and of solutes in the droplet; their inclusion would affect the droplet size distribution, but it is shown in Appendix 2 that the effects are slight compared with the changes brought about by the adiabatic cooling via the supersaturation term. The release of latent heat affects the droplet temperature, but a suitable value of F in Eq. (2) will take care of this last effect.

A useful parameter for describing the progress of condensation by adiabatic expansion is the total mass of droplets in unit mass of air, which will be called the ' liquid mixing ratio ' (L.M.R.), denoted by M and measured in grams per kilogram. Howell (1949) finds that in cloud the supersaturation is less than 1 per cent and varies slowly, so that the increase in M can be found from the decrease in saturated vapour mixing ratio to a good approximation. Since $M = \sum_1 m_r$, the sum of droplet masses in unit mass of air,

$$\frac{dM}{dt} = \sum_1 \frac{dm_r}{dt} = F S \sum_1 r \quad . \quad . \quad . \quad . \quad (3)$$

where $\sum_1 r$ is the sum of the radii of the droplets in unit mass of air. Then

$$\frac{dm_r}{dt} = \frac{r}{\sum_1 r} \frac{dM}{dt} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

or, over a small time interval Δt,

$$\Delta m_r = (r / \sum_1 r)\, \Delta M \quad . \quad . \quad . \quad . \quad . \quad (5)$$

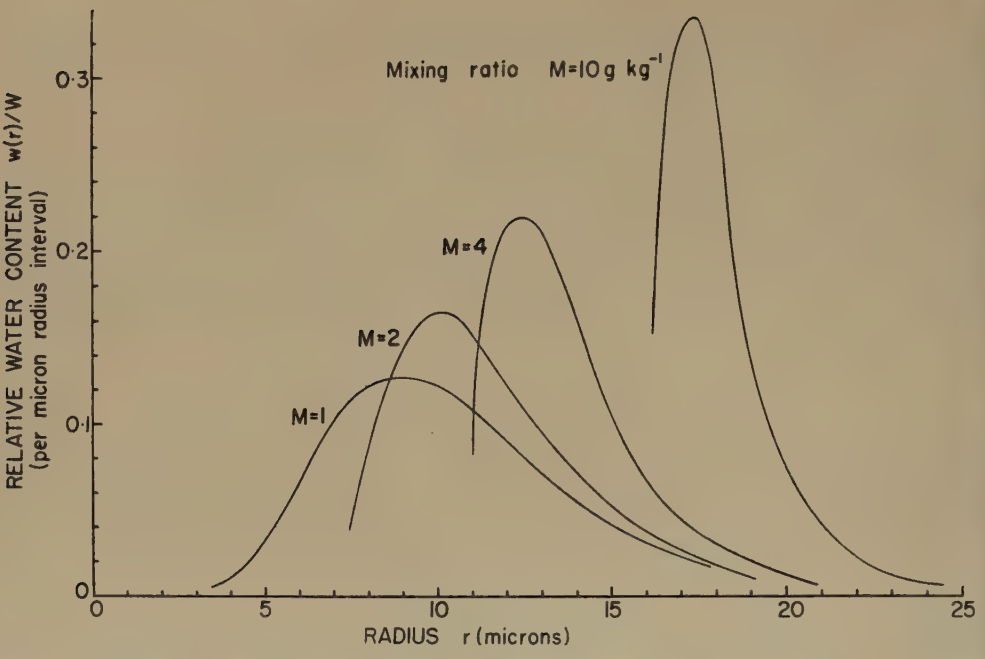


Figure 3. Modification of water-content distribution by condensation. The distribution at $M = 1$ is assumed to be the same as in fair-weather cloud : the other curves show the distribution after water is condensed on to it rapidly. All are normalised to have equal area : the peak water content $w(r)_{max}$ actually increased 26 times from $M = 1$ to 10 g/kg.

Using this equation, an idealized fair-weather distribution ($M = 1$ g/kg) was subjected to successive increments of ΔM of about 25 per cent (beyond $M = 2{\cdot}5$ g/kg, 50 per cent) and the distribution of radii recalculated at each step. The resulting distributions at $M = 2$, 4, and 10 g/kg are shown in Fig. 3. The radii of the smaller droplets increase more rapidly than those of the larger droplets; although dm_r/dt goes as r, dr/dt goes as r^{-1}. In an updraught of 3 m/sec, an air parcel starting from a condensation level at 850 mb and 15°C would take 3, 5, 10½ and 29 min to reach the liquid water mixing ratios 1, 2, 4 and 10 g/kg, if not diluted by entrainment.

(*b*) *Coalescence*

In an encounter between a drop, radius s, and a smaller droplet, radius r, the Langmuir collision efficiency depends critically on the parameter K which is proportional to r^2 and to U/s, where U is the relative velocity. The effect of condensation on U has been slight. Indeed, it might be expected that narrowing the distribution would actually decrease U, but the square law dependence of fall velocity on radius counteracts this tendency. (It happens, in fact, that at every stage in the condensation process $d(s^2)/dt = d(r^2)/dt$ so that the relative velocity between s and r is unaffected so long as Stokes's Law holds for both). The most important effect, so far as collision efficiency is concerned, is that on r, which enters into K as the square. Condensation has in fact increased r more than s (the smallest value of r increases from 4 to 11 μ) and has thereby greatly improved the chance of collisions.

Starting from curves such as those in Fig. 3, the effect of coalescence on each was calculated by the method outlined in § 2 (*b*), but with a new boundary radius r_1, in each case. Fig. 4 shows, for example, the calculated development of the curve for 4 g/kg. After

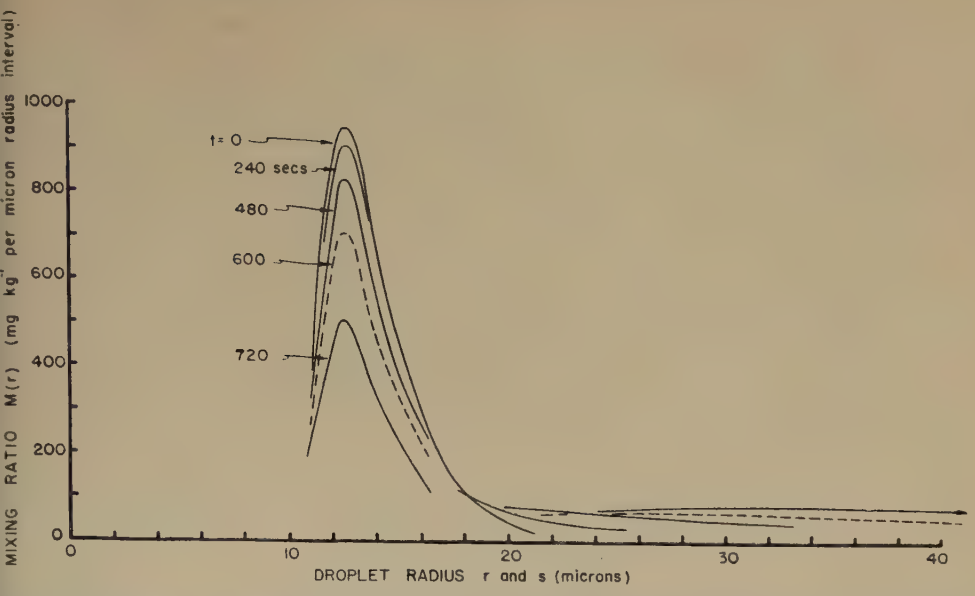


Figure 4. Modification of water-content distribution by coalescence (after condensation). The curve $t = 0$ is the distribution for $M = 4$ of Fig. 3. Droplets, whose radii in the fair-weather cloud lay below 13 μ and whose radii now extend up to 17 μ, are swept up by drops above 17 μ. After 12 min (last curve) the largest drops reach 50 μ.

12 min a precipitating type of distribution has resulted. (In reality, even this short time is overestimated for reasons noted earlier.) The final curve in Fig. 4 represents a fairly advanced stage. The largest drops have a radius of 50 μ which is increasing at the rate of 6 μ/min. The radar reflectivity, as computed from $\sum_1 D^6$, is such that it would give a discernible echo on an AN/CPS-9 weather radar working at full efficiency at 30 mi range, so from there on its progress could be followed by radar. The reflectivity is increasing at the rate of 3 db/min, so that detection on a less powerful radar would only be delayed by a few minutes. Collisions among the larger drops neglected in Fig, 4. have in fact become nearly as important as those between larger and smaller droplets, so a raindrop distribution could be expected to develop shortly.

Continuing for the moment to regard the condensation process and gravity-induced coalescence as acting separately, Fig. 5 shows the rate of growth by coalescence of the largest drop only as a function of its radius s_{max}, for four fixed values of M. Evidently, while ds_{max}/dt is a very sensitive function of its radius, as would be expected, it is also sensitive to liquid mixing ratio M. At $M = 4$ g/kg, when the largest droplet is still only 21 μ radius, it is growing by coalescence at 0·9 μ/min, a faster rate than it would have had even at 40 μ with $M = 1$.

(c) *Condensation and coalescence combined*

When a parcel of cumulus cloud is in an updraught, condensation and coalescence occur simultaneously. The relative importance of these two processes at any stage depends on the updraught velocity; a rapid updraught speeds condensation relative to coalescence. Fig. 6 shows the growth of the largest drops in a parcel lifted by an updraught at various speeds. The cloud base is at 830 mb and 6°C and the parcel is assumed to have the ' fair-weather ' drop-size distribution 580 m above cloud base where $M = 1$ g/kg.

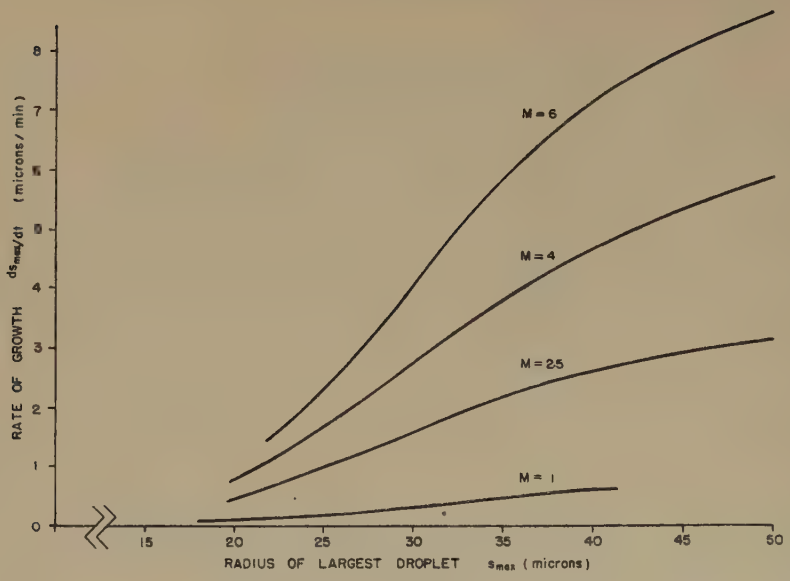


Figure 5. Rate of growth of the largest droplet in a parcel of cloud air as a function of droplet radius and l liquid mixing ratio M (marked on curves in g/kg).

In computing these curves, the detailed development of the drop-size distributions (as in Figs. 2 and 4) was not recalculated; instead, the approximation was made that the rate of growth of the largest droplet (ds_{max}/dt) depends only on its radius, s_{max}, and the liquid mixing ratio, M. The rate of growth was obtained by adding the rate of growth by condensation (not shown, but obtained from curves such as Fig. 3) to the rate of growth by coalescence (Fig. 5); the radius is found as a running time integral, evaluated numerically.

In the result, the time to reach 50 μ is long for slow updraughts and short for rapid ones, so that the liquid mixing ratio, M_{50}, at which this size is reached is not very different. At low velocities more time is available, but most of it is spent at small values of M at which growth is slow; at high velocities the growing droplets are carried somewhat higher to a region where they can complete their growth very rapidly. Table 1 shows that a tenfold variation in updraught velocity produces less than a twofold variation in M_{50}. Updraught velocities in growing cumulus lie between about 1 and 10 m/sec (Browne, Palmer and Wormell 1954) but in clouds in which the updraught is held as low as 1 m/sec for long periods, entrainment of environmental air may well prolong the development even further. In the calculations, entrainment was neglected so the height of first appearance of 50 μ droplets will correspond more nearly to that for more rapid updraughts.

TABLE 1. TIME FROM ENTERING CLOUD BASE TO FIRST APPEARANCE OF 50 μ DROPLET IN A PARCEL, AND LIQUID MIXING RATIO AT THIS APPEARANCE (ascent on a saturated adiabat from base at 6°C and 830 mb)

Updraught velocity (m/sec)	1	3	10
Time t_{50} (min)	36·4	19·2	10·5
L.M.R. M_{50} (g/kg)	3·5	5·1	6·8
Height above cloud base (km)	2·2	3·5	6·3

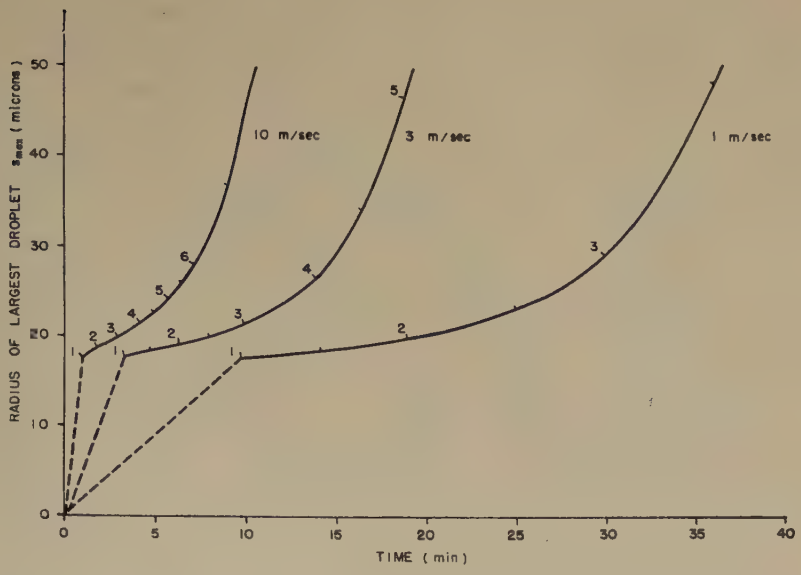


Figure 6. Growth of the largest droplet in a parcel of cloud air as it ascends from cloud base at a steady vertical velocity of 1, 3, or 10 m/sec. Markings on the curves show liquid mixing ratio M in g/kg. Growth from cloud base to the level at which $M = 1$ g/kg was not computed, but was assumed to result in a fair-weather spectrum at $M = 1$.

(*d*) *Experimental verification*

Workman and Reynolds (1949) made measurements of the average height of the initial radar echo in thunderstorms in New Mexico. They give sufficient data to show that at the initial radar echo the liquid mixing ratio was 6·0 g/kg (on the average), and that the ascent to that height must have taken at least 15 min.

Bowen (1951) reported two radar observations in cumulus near Sydney, Australia. From the accompanying data, the L.M.R. at the first radar echoes can be calculated to be 4·6 and 4·3 g/kg. The use of airborne radars at very close range (leading to earlier detection of the droplets) would tend to result in small values. Battan (1953) found that the average height of the top of the first radar echo above the cloud base in Ohio thunderstorms was 10,500 ft. Taking the mean sounding for the atmosphere in Ohio at the time of the radar observations (Braham 1952), it can be shown that the L.M.R. at this level would be 6·5 g/kg. In all these calculations, ascent of a parcel on the saturated adiabat without dilution has been assumed. (In a wide cloud, the core would be diluted very little.)

Battan, Byers and Braham (1955), combining radar and visual observations, found, for three locations, the size of cloud which has a 20 per cent chance of showing a radar echo (larger clouds had, of course, a greater chance). The L.M.R. that an undiluted parcel would have acquired in ascending to the top of such a cloud (initial echo levels are not known, but would lie close to these heights) is 4·1 g/kg for Puerto Rico, 7·0 for Missouri, and 8·8 for New Mexico. The tall clouds in New Mexico may be rather susceptible to entrainment of the surrounding dry air, so that the actual L.M.R. may well have been considerably less than this figure.

The foregoing examples are summarized in Table 2. The evident grouping of values (all but one lie between 4·1 and 7·0 g/kg) of a parameter which in cumulus varies between 0·1 and 8 g/kg roughly, seems to bear out the contention that liquid-water mixing ratio is the most critical parameter for precipitation in these clouds.

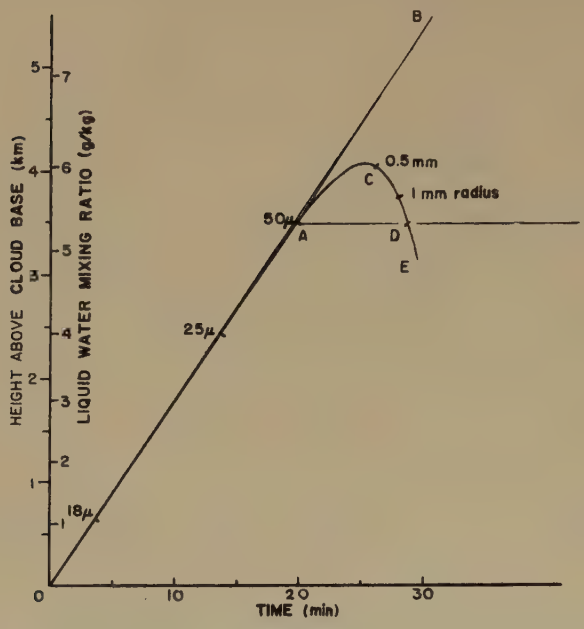


Figure 7. Trajectory of growing droplet. Height above cloud base versus time since leaving cloud base. OB : Air parcel ascends at 3 m/sec. OACDE : Behaviour of largest droplet in the parcel as it grows by condensation and coalescence together (radius marked on curve). A fair-weather droplet-size distribution is assumed at the point '18 μ.' AD : Later parcels have also produced 50 μ droplets when they reach this line. The region BADE contains radar-echoing material.

TABLE 2. LIQUID MIXING RATIO AND TEMPERATURE AT THE INITIAL RADAR ECHO, OR AT THE TOP OF JUST-PRECIPITABLE CLOUDS

Author	Year of publication	Locality	L.M.R. (g/kg)	Temp. (°C)
Workman and Reynolds	1949	New Mexico	6·0	− 9
Bowen	1951	Sydney, Australia	4·6	+ 4
			4·3	+ 4
Battan	1953	Ohio	6·5	+ 1
Battan, Byers and Braham	1956	Puerto Rico	4·1	+ 11
		Central U.S.	7·0	− 3
		New Mexico	8·8	− 15

4. DEVELOPMENT OF PRECIPITATION

So far, the properties of a single parcel of cloudy air have been discussed. In this section these properties are related to the structure of a rain cell.

(*a*) *Trajectories in the updraught*

First, the rise and fall of the largest droplets in an updraught of 3 m/sec was calculated, making the same assumption as in § 3 (*c*) that ds_{max}/dt depends on M and s_{max} but not on the detailed shape of the droplet size distribution. Beyond 50 μ radius, growth by condensation was neglected and an average value of collision efficiency and an average value of liquid mixing ratio were taken.

The growth of the largest drops, and the distance they fall relative to the air, were hen computed by numerical integration. Fig. 7 shows the variation of height with time trajectory) of the largest drop in a parcel which started from cloud base at zero time and vas carried up at 3 m/sec steadily. In 20 min the drop grows to 50 μ and lags only 82 m elative to the air (point A). In another 5 min, however, it reaches its maximum height of 4·1 km and starts to fall toward the ground (ACE), though the smaller drops in the parcel are carried higher and fall later. Parcels entering the cloud base later than this one give identical, but delayed, drop trajectories.

Bowen (1950) – the relevant figure is reproduced as Fig. 1 of Browne, Palmer and Wormell (1954) – assumed an updraught of 1 m/sec, a constant supersaturation of 0·1 per cent and a cloud having constant water content of 1 g/m^3 and constant (and monodisperse) droplet radius of 10 μ; he found that a drop would be carried to a maximum height of 2·2 km above the level at which it was 13 μ radius and would take 45 min to do so. If he had taken 3 m/sec for the updraught, surely not an excessive value, the rise would have been 6·6 km. Our more realistic assumptions (in which supersaturation, water content, and droplet-size distribution all vary) lead to a rise of 4·1 km from cloud base (3·3 km from the 18 μ level) at this updraught velocity. A further increase in velocity gives a less than proportionate increase in maximum height, since growth by coalescence is quicker at higher levels.

(*b*) *The initial radar echo*

The parcel described in the foregoing discussion would first become visible on a radar at the time and height A (Fig. 7), because the development which produces 50 μ drops has also brought the reflectivity up to detectable level, or so the calculations showed. Though the largest drops would fall out (CE) smaller drops which stayed with the rising air would grow to detectability, and the top of the echo would follow the locus AB. Subsequent parcels would rise along parallel trajectories, giving detectable echoes at the same levels but at later times.

If all parcels ascended without interruption from the base, the lowest position of the echo would always be at height A, until droplets grown earlier fell back to this level. The bottom of the echo, then, would follow ADE. This is contrary to most observations. Battan (1953) found that the radar echo when first seen averaged 1,700 m deep (instead of a point as at A) and that its lowest point descended rapidly during the next few minutes. (The statistical concept of Telford (1955) introduces some variability into the times at which drops mature, and would blur the outlines of the echo, but not sufficiently to account for the observations.)

The Reynolds number of flow in an active cumulus, however, is of the order of 10^7, so that the updraught is unlikely to be a purely laminar flow. Then not all parcels of air rise without interruption, but some are exchanged with the surrounding cloud air; having risen to some particular level, they then remain at that level. (If, as Scorer and Ludlam (1953) suggest, the updraught consists of ' bubbles,' the ' wake ' consists of air which rose inside bubbles and was eroded at that level.) In parcels which have their upward motion halted in some such way, coalescence will still cause the droplets to grow to 50 μ radius, but this will take somewhat longer, depending on the liquid mixing ratio that has been reached. From the results of the condensation-coalescence study, the times taken to reach 50 μ by parcels which left the updraught at various levels were calculated; they are plotted in Fig. 8 and joined up by the locus AF. Bearing in mind that a parcel is first detected when its largest droplets reach 50 μ (according to § 3 (*b*)), the locus shows the initial great depth and subsequent rapid descent, characteristic of the first radar echo.

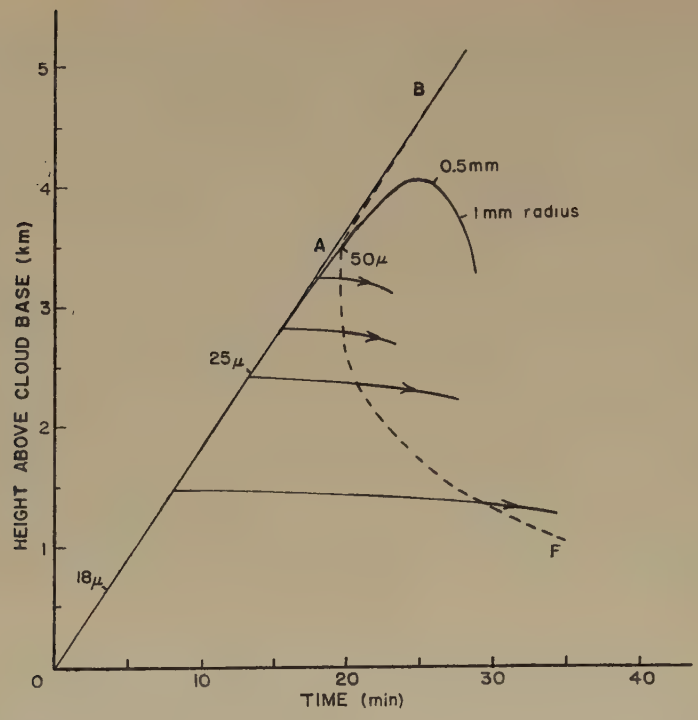


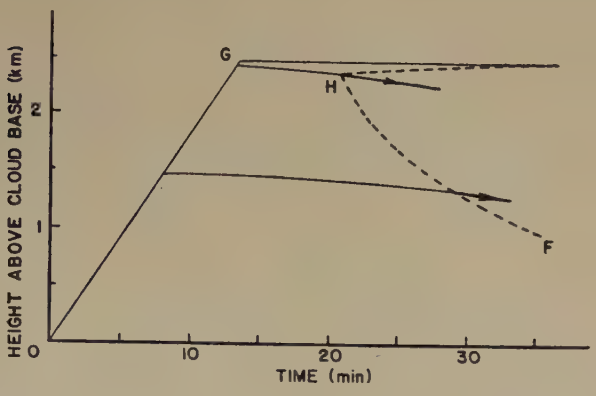
Figure 8. Development of initial radar echo. The main parcel rises steadily at 3 m/sec (OB) but at each of a number of levels a portion leaves the parcel and remains at that level. The largest droplet in each portion grows and falls slightly as shown. The locus AF joins up points at which the first 50 μ drop appears in each portion. The region BAF contains radar-echoing material. At 20 min, radar echo appears over a great depth almost simultaneously.

Battan foresaw that it is not necessary to postulate any rapidly descending particles in the initial radar echo. His analysis led him to remark (Battan 1953, p. 323) : ' The alternative explanation of the *initial* lowering of the echo, involving the growth of drops to detectable sizes at successively lower layers of the cloud, seems reasonable. It was found that the first echo had an average depth of about 5,700 ft, with only 11 per cent less than 3,000 ft, and consisted of two parts in at least one case. Of course, the depth of the echo was, to a certain extent, a function of the time between observations. This indicates that droplets in a fairly large region of the cloud attained detectable size at about the same time. It is suggested that the lower limit of this region of droplet growth descends with time.'

(c) *Arrested development*

It often happens that, owing to the presence of an inversion (or some such condition of the surrounding air), the rising cloud air loses its buoyancy and spreads out horizontally. The echo will then (ideally) behave as shown in Fig. 9, where the air stops rising at G. The initial appearance of the echo, at H, is delayed, and occurs at a lower mixing ratio than A : the echo top does not rise, though the bottom descends (HF). The cloud depth may be so small as to delay the appearance of the echo considerably.

On the other hand, the parcel which comes to rest at the cloud top is likely to become mixed with unsaturated environmental air. According to Malkus (1955) such a parcel acquires negative buoyancy, starting a downdraught in which evaporation takes place.



igure 9. Development in an arrested cloud. The main parcel stops rising at G. The largest droplet in follows GH. The locus HF joins up points at which the first 50 μ drops appear at various levels. The region to the right of HF contains echoing material.

Conditions in a downdraught are less favourable to growth by coalescence, and if carried down far enough even the largest droplets may evaporate. If a general downward motion is set up, the process sketched in Fig. 9 may never be completed : possibly this accounts for the lack of echo from cumulus humilis (Plank, Atlas and Paulsen 1955). The exact course of coalescence in a non-precipitating cloud requires further study.

5. Summary and conclusions

In cumulus clouds which develop precipitation, it is observed that (i) liquid-water content increases as the cloud matures. There must be continued condensation from the vapour. (ii) Total droplet population decreases as the cloud matures. Coalescence must be taking place. (iii) The droplet-size distribution changes radically, from one lying entirely below about 18 μ radius, to one extending up to 100 μ or more. An appreciable fraction of the droplets must be growing quite rapidly.

Near the cloud base, differential settling is unable to produce the observed development because the smaller droplets are able to avoid collisions easily. Higher up, where the droplets are enlarged by condensation, collisions occur readily and a heavy cumulus type of distribution quickly results. Until the cloud builds up to a high enough level, coalescence is too slow to develop precipitation. Precipitable droplets appear at the level at which the liquid-water mixing ratio has reached between 4 and 7 g/kg, depending somewhat on updraught velocity. Observations of the limiting height of precipitable clouds and of the height of the first radar echo agree with this result. If it is true that updraughts are not laminar but subject to exchange with the surrounding cloud, the mechanism of condensation followed by coalescence can also account for the observed behaviour of the initial radar echo.

The present study of convective precipitation has not considered in detail the role of ice-crystal growth, nor the effect of salt on droplet sizes. Rapid ice-crystal growth is confined to temperatures between about $-10°C$ and $-20°C$; Table 2 suggests that in many parts of the world conditions in cumulus are not favourable. Though it is entirely possible for the sea-salt mechanism (Ludlam 1951, Woodcock 1952) to be acting concurrently with the condensation-coalescence mechanism described, the latter seems better able to account for the observed droplet-size distributions. It constitutes a basic process which must always be active in sufficiently deep clouds.

Acknowledgments

The author wishes to thank Professor J. S. Marshall for his advice and encouragemer throughout the work. Thanks are also due to Drs. Ross Gunn and Heinz Lettau fo assistance on various points. The work reported in this paper was sponsored by th Geophysics Research Directorate of the U.S. Air Force Cambridge Research Center.

Appendix 1

Collection efficiency

In applying Langmuir's (1948) work to the problem the following assumptions wer made :

1. Langmuir's theory, worked out for droplets much smaller than the drop, is sti applicable when the radii are more nearly equal.
2. It applies despite appreciable motion of the droplets through the air, if use i made of the relative velocity U.
3. The theory also applies when the velocities are not steady.
4. Collisions always lead to coalescence. Justification for this was discussed by Eas and Marshall (1954).

These assumptions are carried over into the present work.

The efficiency E (corresponding to Langmuir's E_v), is always less than unity, an can be zero. It is a unique, monotonic function of a dimensionless parameter $K = U\tau_r/$ where $\tau_r = 2\rho_s r^2/9\eta$; the equation is

$$E = 0, \qquad K \leqslant 1{\cdot}214$$

$$E = \left[1 + \frac{\frac{3}{4}\ln(2K)}{K - 1{\cdot}214}\right]^{-2}, \qquad K > 1{\cdot}214 \quad . \quad . \quad . \quad (6$$

(This relationship is reproduced in Fig. 1, curve E).

Appendix 2

Approximations used in the condensation theory

Howell (1949) has described the process of condensation in ascending air. The ai is cooled until water vapour condenses on to nuclei, making them into droplets; the vapour then continues to condense on to the droplets. The air becomes slightly super-saturated and a droplet gains water according to the following law :

$$\frac{S}{100} = Fr\frac{dr}{dt} + \frac{F_1}{r} - \frac{F_2}{r^3} \quad . \quad . \quad . \quad . \quad (7)$$

where r = radius of droplet

S = percentage supersaturation

F_1 is defined by Howell and takes account of the work done against surface tension,

F_2 depends on the soluble material in the nucleus,

$F = F_4 + F_5 F_6$, where F_4, F_5, and F_6 relate to the transport of vapour and heat.

McDonald (1953) claims that Howell used an incorrect value of F_2 through neglecting o consider Raoult's Law and assuming 100 per cent dissociation of the electrolyte when a ygroscopic nucleus is dissolved. Had allowance been made for incomplete dissociation, he calculated relative rates at which nuclei of various sizes grow into droplets would have een considerably affected. This in turn would have left its mark on the results of all ubsequent calculations of droplet size distributions. Whether this is the case or not, the ize distribution of condensation nuclei is not known with any certainty so no great reliance an be placed on curves of droplet sizes derived from them. Both difficulties are avoided y starting from the idealized fair-weather cumulus distribution, which is an approximation o observed droplet distributions. All the droplets in it are large enough for the F_2 term o be quite negligible compared to the others. At updraught velocities of 1 m/sec or nore, dr/dt is sufficiently large for the first term to predominate and the F_1 term can lso be neglected.

So far, the theory has been derived in terms of adding vapour to the general vapour ield in a parcel of air at constant volume. In a cloud the process is, of necessity, one in vhich the volume increases. The findings are still applicable, however, if liquid water nixing ratio M is substituted for water content W, at least to the present degree of pproximation. The expansion of the air parcel, in its effect on coalescence times, is llowed for when the M distribution is converted back to a W distribution.

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A theoretical estimate of the collection efficiencies of small droplets

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Summary

The effective collision cross-sections between pairs of small water droplets, falling freely through air under the influence of gravity, have been computed on the assumption of hydrodynamical effects alone being important. It has been assumed that each droplet tends to fall freely at its terminal velocity with respect to the surrounding medium, which also moves under the influence of the motion of a neighbouring droplet. By the use of previously computed fields of flow of a viscous medium around a uniformly translated sphere, the trajectories of motion of one droplet with respect to another were computed for a range of droplets having radii between 4 μ and 200 μ.

It is found that the collision cross-sections increase to values considerably greater than the corresponding geometrical cross-section as the droplets approach equality of size. As the difference in size increases, the collision cross-section decreases to values close to the geometrical cross-section, and finally decreases to zero. For any one particular size of the collecting droplet there is a minimum size of collected droplet below which no collection is possible. This size decreases with increasing size of the larger droplet greater than 10 μ in radius. For droplets smaller than 10 μ in radius, collision can occur only between droplets of nearly equal size. The results could account for coalescence becoming an important factor in the rapid formation of raindrops for droplets of radii greater than 10 μ.

1. Introduction

It is believed that cloud droplets, initially developed by a process of condensation, are mostly of sizes lying within the range 5 to 10 μ in radius together with a small proportion of droplets of greater size. It is considered that further increase in size toward the formation of raindrops must take place by a process of coalescence.

A theoretical estimate of collection efficiencies, or effective collision cross-sections for coalescence of droplets, has previously been made by Langmuir and Blodgett (1945), Langmuir (1948), and their results have been used by some workers in attempts to calculate the expected rate of coalescence and increase in size of droplets in non-freezing clouds. It appears, on the basis of Langmuir's figures, that a period of an hour or more must elapse before precipitation can occur (Bowen 1950). The actual time before precipitation occurs, although very variable, is sometimes considerably less than this estimate. Thus, either collection efficiencies are greater than those given by Langmuir (1948), or relatively large droplets are generated by some process other than coalescence, from which condition coalescence may proceed rapidly; or other, as yet unconsidered, factors are important in enhancing the coalescence process. This agrees with some experimental evidence (Adderley 1953).

It is here shown that collection efficiencies, estimated upon the basis only of hydrodynamic interaction between droplets, can become much larger than those quoted by Langmuir (1948), particularly between droplets of nearly equal sizes. This increase is due to the formation of a wake behind a moving droplet within which there is a strong tendency for the fluid to follow the droplet. In this respect the theoretical results agree with those of coalescence experiments carried out by Telford, Thorndike and Bowen (1955).

As the size of the collected droplet decreases, the collection efficiency of the collecting droplet decreases to values below that given by the geometrical cross-section which would apply if no hydrodynamic forces were involved. Eventually a droplet size is reached below which no collision or coalescence occurs.

Allowance has been made in the calculations, as far as is currently possible, for effects due to the variation of the rate of fall of droplets under gravity, changes of the flow pattern of the air around a droplet for different velocities, and the forces acting upon droplets due to the interaction with the flow patterns. The first of these factors allows for variation of rate of approach of droplets as a function of their sizes, the second allows for the transition of the flow patterns from that of Stokes's type, applicable to droplets of radius less than 10 μ, to that approaching flow with a well developed boundary layer and a strong wake. Changes in the drag force acting upon a droplet within the field of flow of another cause the droplets to interact and traverse curved trajectories.

It has been assumed that all droplets fall through perfectly still air; effects arising from atmospheric turbulence have not been allowed for.

The calculations cover a range of sizes of collecting droplets varying from 4 μ to 200 μ in radius. The lower part of this range includes the region where it is thought growth by condensation is replaced by growth by coalescence. The collision cross-sections calculated are shown in graphical form in Fig. 1, in which the ordinate represents the linear collision cross-section in units of the radius of the larger, or collector, droplet and corresponds to the square root of the collection efficiency, E, as defined by Langmuir.

2. Viscous flow around spheres

At sufficiently low velocities, or small Reynolds numbers, the flow of a viscous fluid around, and in the neighbourhood of, a uniformly translated sphere corresponds closely to that given by Stokes. The velocities of such a flow decrease with distance, r, from the sphere, as r^{-1} only. At great distance, however, inertia forces within the fluid are more

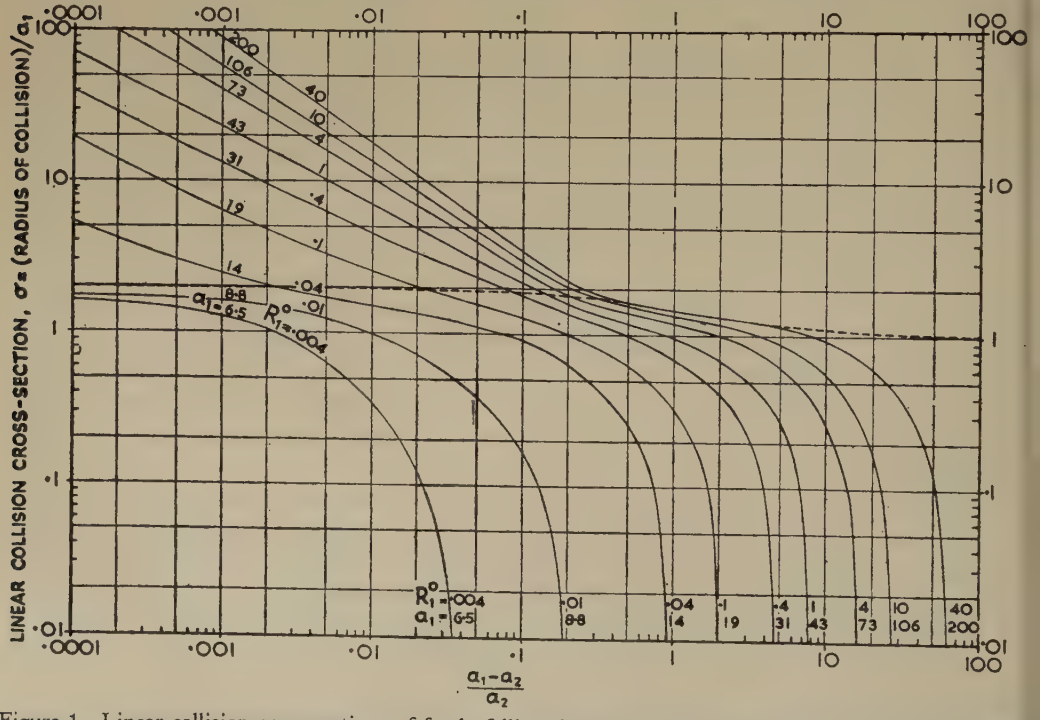


Figure 1. Linear collision cross-sections of freely falling droplets. A droplet of radius a_1 microns, falling freely through air under gravity, collides with droplets of radius a_2 microns ($a_2 \leqslant a_1$) lying within a horizontal distance σa_1 from its axis of motion. The full lines show σ, as a function of $(a_1 - a_2)/a_2$, for various values of a_1. The dotted line corresponds to the geometrical cross-section, $(a_1 + a_2)/a_1$. The droplets are assumed to approach from infinite separations.

portant than viscous forces, and the flow differs considerably from that of Stokes. s the velocity of the sphere increases the flow pattern rapidly changes its character, and o regions of essentially different types of flow become distinguishable, one in which the w is irrotational and a second, the wake, in which the flow is predominantly rotational.

Exact knowledge of the flow pattern of viscous fluid around a uniformly translated here for Reynolds numbers of the order of unity and greater does not yet exist. An act determination of these patterns would be a task of considerable magnitude, and e of such information in calculations such as those reported here, would render the sults only little more realistic.

The difficulty arising from the non-linear character of the equations of fluid motion ay be largely overcome, and the main properties of the flow pattern described, by opting the well-known approximation due to Oseen. Solutions according to the approxi- ation of the case of the uniformly translated sphere have been given by Goldstein 929) and have recently been computed in detail by Pearcey and McHugh (1955).

The solution according to Oseen's approximation satisfies all the necessary boundary nditions and generally possesses the correct asymptotic form at large distances from the here, although it deviates from the actual flow by some quantity varying linearly with eynolds numbers at moderate distances from the sphere. In the absence of more adequate formation of the flow patterns, those already computed according to this assumption ve been extended to $R = 40$, and adopted in the present calculation.

These flow patterns give values of the drag force acting upon the sphere which agree asonably well with the experimentally determined values up to $R = 10$; above this lue the solution tends to overestimate the strength of the wake and the drag. Although all cases the experimental values of the drag force were used, the results for the case = 40 should be considered only as indicating a trend in the behaviour of the collection ficiencies.

At points close to the sphere the expressions according to Oseen's approximation r the flow velocities are very complicated but at great distances they may be expressed follows,

$$\left.\begin{aligned} u_r &= \frac{A_0}{r^2} - \frac{\pi}{Rr}\exp\left(-Rr(1+\mu)/4\right)\sum_{n=0}^{\infty} B_n(1-\mu^2)P_n(\mu) \\ u_\theta &= \sin\theta\left\{\frac{A_1}{r^3} - \frac{\pi}{Rr}\exp\left(-Rr(1+\mu)/4\right)\sum_{n=0}^{\infty} B_n P_n(\mu)\right\} \end{aligned}\right\} \quad . \qquad (1)$$

here u_r and u_θ are the velocity components referred to the velocity of the sphere in the dial and angular directions respectively; R is the Reynolds number of the sphere (equal diameter × velocity/kinematic viscosity). P_n refers to the Legendre function of order and $\mu = \cos\theta$, θ being the angle between the radius vector and the forward axis of otion. The distance r is measured in units of the radius of the sphere, and A_0, A_1, d B_n are functions of R.

Within the wake, for which θ is close to π, the velocity components approximate to

$$u_r = -\frac{RA_0}{2x}e^{-Ry^2/8x}, \qquad u_\theta = u_r\frac{y}{2x} \quad . \quad . \quad . \qquad (2)$$

here x is the distance measured backward along the previously traversed path, and y the normal distance from the path. The wake occupies a roughly paraboloidally shaped gion, enclosing the traversed path, within which the motion of the fluid is directed mainly ward the sphere and inward toward the axis. In addition to the exponential decrease velocity away from the axis there is also a further decrease with distance as x^{-1}. As e velocity of the sphere increases, the effective width of the wake decreases as $R^{-\frac{1}{2}}$, hilst the absolute velocity of the fluid at distance xa to the rear increases as $R^2 A_0$.

TABLE 1. VALUES OF PARAMETERS DETERMINING THE FLOW PATTERN AROUND A UNIFORMLY TRANSLATED SPHER

R	A_0	A_1	RA_0	$R^2 A_0/2$	RA_1
0·0	∞	0·500	3·000	0·0	0
0·1	30·188	0·500	3·019	0·1509	0·0500
0·4	7·688	0·4875	3·075	0·6150	0·1950
1·0	3·526	0·4508	3·526	1·763	0·4508
4·0	1·2175	0·3793	4·870	9·740	1·5172
10·0	0·7169	0·3276	7·170	35·85	3·276
40·0	0·51	0·28	20·4	400	11·2

Outside the wake where the exponential terms in the velocity are small, th components approximate to,

$$u_r = A_0/r^2, \qquad u_\theta = A_1 \sin\theta/r^3 \quad . \quad . \quad . \quad (3$$

so that at sufficiently great distances $u_r \gg u_\theta$, and the motion is predominantly radia Further, in this region, u_r decreases with distance more rapidly than within the wak The field velocities vary with R as RA_0 and RA_1. From Table 1 it is seen that whil A_0 and A_1 decrease with increasing R, even for R as large as 40, both A_0 and A_1 are c the same order of magnitude. Hence the flow outside the wake, although irrotationa differs considerably from the ideal aerodynamic flow in which A_0 would be zero an $A_1 = \frac{1}{2}$. However, as R increases, the values of A_0, A_1 etc., given by Oseen's approxima tion may be expected to differ considerably from those which actually apply, and thi difference becomes important in the neighbourhood of $R = 40$ and beyond.

Close to the sphere, where the fluid tends to take up the velocity of the sphere, th tendency with increasing R for a boundary layer to form becomes pronounced, and rapid transition of the flow of the medium from a mainly radial to an axial direction occur close to the surface.

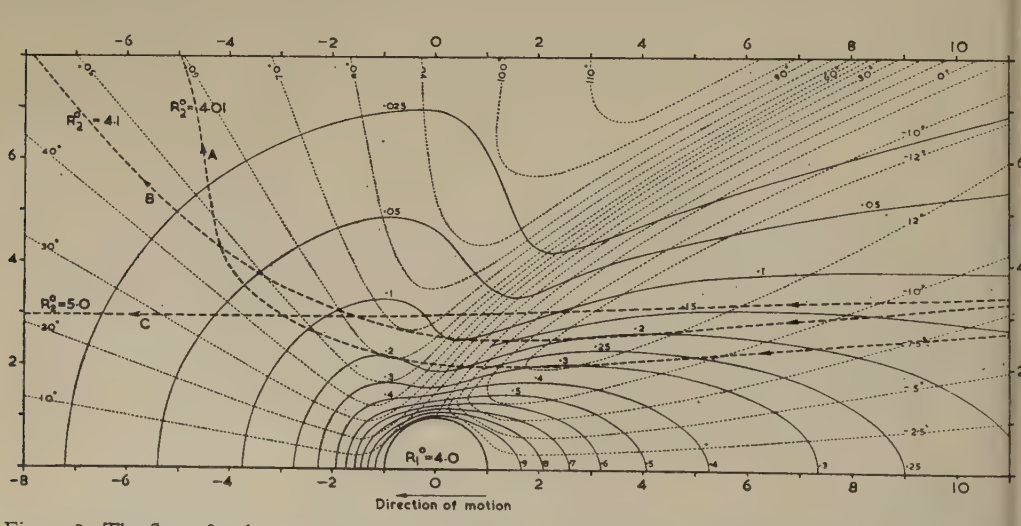


Figure 2. The flow of a viscous fluid around a falling droplet. The figure corresponds to the case of a wate droplet of radius 73 μ (Reynolds number 4·0) falling freely in air. As shown, the droplet is assumed moving from right to left. The full lines indicate contours of constant magnitude of fluid velocity relative to the speed of the droplet. The lighter dotted lines show contours of constant direction of fluid motion; the numbers indicate the angle between the direction of flow and the forward axis of motion.

The heavy dotted lines show typical trajectories followed by slightly heavier droplets which just avoid collision, and A, B and C, correspond to $a_1 - a_2/a_2$ equal to 0·001, 0·011, and 0·109 respectively. They show the 'suction' effect within, and the repulsive effect outside, the wake.

Detailed computations of the flow patterns were carried out in the cases $R = 1$, 4, d 10 (Pearcey and McHugh 1955) and approximately for $R = 40$. Figs. 2 and 3 show typical flow pattern, for the case $R = 4$, in which all the properties mentioned e illustrated. Fig. 2 shows the pattern close to the sphere, and Fig. 3 illustrates the aracter of the wake. These flow patterns were used in the computation of the collision oss-sections. For cases of small R (< 1) analytical expressions were used for the flow locities which reduce approximately to

$$\left.\begin{aligned} &= \frac{3\mu}{2}\left(\frac{1}{r} - \frac{1}{3r^3}\right) - \frac{3}{2r}\left(\mu + \frac{1-\mu}{2}\, e^{-Rr(1+\mu)/4}\right) + \frac{3}{Rr^2}\left(1 - e^{-Rr(1+\mu)/4}\right) + 0\,(R^2) \\ &= \sin\theta \left\{\frac{3}{4}\left(\frac{1}{r} + \frac{1}{3r^3}\right) - \frac{3}{4r}\,(1 - e^{-Rr(1+\mu)/4})\right\} + 0\,(R^2) \end{aligned}\right\} \quad (4)$$

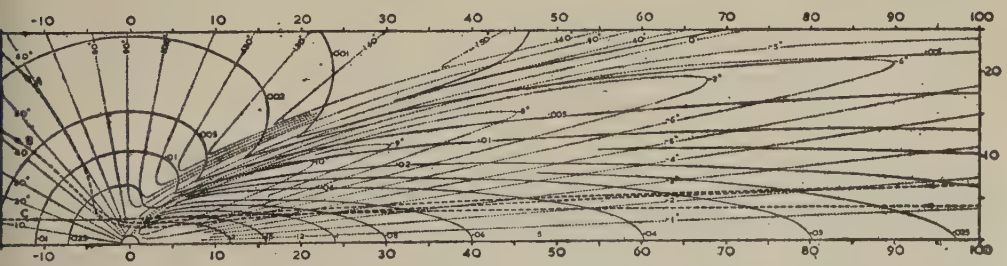
The first component of each of these expressions corresponds to the flow in the limit hen $R = 0$, to Stokes flow; the remainder expresses the asymmetry in the flow which ccurs as soon as R becomes non-zero. Thus, Stokes flow applies from the surface of the here up to distances of the order $4/R$. At larger distances the latter terms have the effect reducing the components to the previous expression (1) in which A_0 and A_1 are equal $3/R$ and $\frac{1}{2}$ respectively and terms $n > 0$ are neglected. The asymmetry introduced by small non-zero R reduces the outward flow to the front of the sphere and augments e inward flow to the rear, which corresponds to the effect of a very weak wake.

3. The equations of motion of neighbouring droplets

In the present estimation, account has been taken of hydrodynamical effects only, nd coalescence has been assumed to take place immediately the surfaces of adjacent roplets make contact. Effects due to possible electrical charges and those depending pon the nature of the surfaces, and of condensation and evaporation have been omitted, also have the possible effects of microturbulence of the atmosphere.

We are concerned with the interaction between pairs of droplets. The larger will be ferred to as the collector, and the smaller as the collected. Variables and parameters ill be accompanied by the suffix, 1 or 2 respectively. A droplet of radius a, and ass m, travels with velocity $\mathbf{V}$, with a Reynolds number R, and under conditions of olated free fall under gravity, moves with a terminal velocity $\mathbf{V}^\circ$ and Reynolds number $^\circ$. Relative velocities between droplets will be denoted by $\mathbf{v}$ and the difference in terminal elocities by $\mathbf{v}^\circ$.

In estimating the relative motions of droplets due to interaction between their spective fields of flow it has been assumed that individual fields are superposable, the elds of total flow being compounded of the sum of the flow patterns of each droplet



gure 3. The flow of a viscous fluid behind a falling droplet. The figure corresponds to the case illustrated Fig. 2, and shows the intensity and shape of the wake to relatively great distances. Flow within the wake generally from right to left and toward the axis of motion. The trajectories, the heavily dotted lines, correspond to those in Fig. 2.

supposed travelling uniformly relative to the fluid velocity at its centre due to the other droplet. Thus $\mathbf{u}_1$ and $\mathbf{u}_2$ are the actual flows which each would produce at the centre of the other, and are related to the velocities of the droplets by the transformations

$$\left.\begin{aligned}\mathbf{u}_1 &= \mathbf{F}_1(\mathbf{V}_1 - \mathbf{u}_2) \,.\, |\mathbf{V}_1 - \mathbf{u}_2| \\ \mathbf{u}_2 &= \mathbf{F}_2(\mathbf{V}_2 - \mathbf{u}_1) \,.\, |\mathbf{V}_2 - \mathbf{u}_1|\end{aligned}\right\} \quad . \quad . \quad (5)$$

where $\mathbf{F}_1$ and $\mathbf{F}_2$ are the vector fields of fluid flow of each droplet as illustrated in Figs. 2 and 3, and are functions of the Reynolds numbers of each droplet considered as vectors:

$$\mathbf{R}_1 = 2(\mathbf{V}_1 - \mathbf{u}_2)\, a_1/\nu, \qquad \mathbf{R}_2 = 2(\mathbf{V}_2 - \mathbf{u}_1)\, a_2/\nu$$

Thus, for example, Fig. 2 shows the vector field $\mathbf{F}$ of a sphere supposed moving with a Reynolds number $R = 4$. To obtain the actual flow at a point in this field with respect to the direction of motion of the sphere, the value of $\mathbf{F}$ at that point must be multiplied by the actual speed of the sphere through the fluid.

By linearly combining both flow patterns, the boundary conditions for the total flow at infinity are satisfied exactly, and those at the surface of the droplets are satisfied approximately. The error introduced depends upon the rate of change of the flow fields $\mathbf{F}_1$ and $\mathbf{F}_2$ and is small for large separations. As the droplets approach one another complex hydrodynamical forces other than those considered here may occur. However, the momentum of the approaching droplet is usually so large as to carry it through such a region of close approach so rapidly that additional effects are unlikely to affect the relative motions appreciably.

It has been assumed that the drag force experienced by a droplet at any instant is the same as that which it would experience when travelling through the medium with uniform velocity. In the computations this force has been taken to be the same as that obtained experimentally by Liebster (1927) and Schmiedel (1928), and agrees with those quoted by Langmuir (1948). The drag force depends upon the Reynolds number of the droplet, and if the actual forces be $\mathbf{D}_1$ and $\mathbf{D}_2$, then

$$\left.\begin{aligned}\mathbf{D}_1 &= -6\pi\nu^2\rho\,\mathbf{R}_1 \,.\, S(R_1) = 6\pi\nu\rho a_1(\mathbf{V}_1 - \mathbf{u}_2) \,.\, S(R_1) \\ \mathbf{D}_2 &= -6\pi\nu^2\rho\,\mathbf{R}_2 \,.\, S(R_2) = 6\pi\nu\rho a_2(\mathbf{V}_2 - \mathbf{u}_1) \,.\, S(R_2)\end{aligned}\right\} \quad . \quad (6)$$

where ρ is the density of the medium and S is a single-valued continuous scalar function of R.

In the case of water droplets in air, flotation forces may be neglected, and when falling freely under gravity the equation of motion of either droplet becomes

$$\dot{\mathbf{V}} = \mathbf{g} - \mathbf{D}/m$$

where $\mathbf{g}$ is the vector of gravitational acceleration, and is related to the terminal velocity of free fall, $\mathbf{V}^\circ$, by

$$\mathbf{D}^\circ = m\mathbf{g} = 6\pi\rho\nu a\,\mathbf{V}^\circ\, S(R^\circ)$$

Table 2 shows the relation between Reynolds number R°, V°, the radius, mass, and the drag parameter S.

A complete solution of the hydrodynamical factors affecting coalescence would require a study first of the detailed flow pattern between droplets at various relative positions and travelling at various speeds; and second of the effects upon the flow fields and the drag forces of the accelerations which must necessarily be experienced by the droplets. These factors have been omitted from this discussion. In many circumstances it may be shown that the effects would be small compared with those allowed for (Pearcey and Hill 1956).

TABLE 2. PHYSICAL PARAMETERS RELATING TO WATER DROPLETS FALLING FREELY UNDER GRAVITY

R	$S(R)$	Radius a (μ)	Velocity $V°$ (cm/sec)	Mass m (μg)
0·004	1·000	6·5	—	—
0·01	1·002	8·8	0·97	0·00244
0·04	1·007	14·	2·44	0·00983
0·1	1·018	19·	4·50	0·0248
0·4	1·073	31·	11·1	0·105
1·0	1·176	43·	19·9	0·287
4·0	1·45	73·	47·	1·42
10·0	1·78	106·	80·	4·34
40·0	3·01	200·	170·	29·4

4. RELATIVE IMPORTANCE OF HYDRODYNAMICAL FACTORS

Clearly a larger droplet can collect only smaller droplets by reason of its superior erminal velocity. Thus, $V_1° \geqslant V_2°$, $R_1° \geqslant R_2°$ and some simplifications may be achieved hen $R_1°$ and $R_2°$ lie within certain ranges. Various cases may be distinguished as follows:

(*a*) *Large droplets*: $R_1° \geqslant 1{\cdot}0$

(i) $R_1° - R_2° \ll R_1°$. In this case both droplets are of nearly the same size, and the arger overtakes the smaller with a relative velocity much less than the terminal velocity f either. Greatly enhanced collision cross-sections occur owing to the drift suffered by he overtaking droplet during its progress through the wake of the smaller droplet.

(ii) $R_1° - R_2° \gg R_2°$. The collected droplet is so much smaller than the collector hat it suffers deflection by the forces arising from the field of flow around the front urface of the latter. The resulting collision cross-section decreases to zero at some pecial value of $R_2°$, depending upon $R_1°$, below which no collision occurs.

(iii) $R_1° - R_2° \simeq R_2°$. When the inertia of both droplets is sufficiently large and heir relative terminal velocities differ widely, hydrodynamical forces do not have sufficient ime to affect the path of fall of either droplet, and the collision cross-section approxiates to the geometrical cross section.

(*b*) *Small droplets*: $R_1° < 1{\cdot}0$

When both droplets are small the fields of flow both forward and backward of both roplets are of nearly equal magnitude and relative velocities of approach are small. Mutual interactions cause reduced collision cross-sections, and the range of sizes of drop-ets which are capable of collision diminishes as $R_1°$ decreases.

5. THE COALESCENCE OF DROPLETS OF NEARLY EQUAL SIZE

When the collector and collected droplets are sufficiently large ($R \gg 1$) and of nearly he same size, $R_1° - R_2° \ll R_1°$, the collector is sufficiently large for a pronounced wake o appear in its flow pattern. It may acquire an enhanced collision cross-section due to s passage along the wake of the smaller droplet.

Collision between droplets of nearly equal size may conceivably occur in two different ays, both depending upon the flows within the wakes. First, collision may occur directly, hat is, the larger droplet may fall and collide with the smaller under the influence of the ake of the latter. Second, the larger droplet may pass the smaller so closely as to engage he latter within its wake, in which it falls sufficiently rapidly to overtake the larger droplet ith which it eventually collides. These cases will be discussed in turn and may be called irect and indirect collision processes respectively.

(a) *Direct collision*

The cross-section of direct collision depends upon the sizes of both droplets through the following factors. The size of the smaller droplet controls the intensity and width of the effective wake, and the difference in size affects the relative terminal velocity and thus the velocity of approach of the larger droplet.

Velocities of flow within the wake increase rapidly with R_2°, i.e., as $R_2^2\, A_0\,(R_2)/2x$ and $R_2^2\, A_0\,(R_2)y/4x^2$, and the width of the wake decreases roughly as $R_2^{-\frac{1}{2}}$. Velocities directed radially outward increase only as $R_2\, A_0\,(R_2)/r^2$.

So long as x is sufficiently large, the effect of the collector on the motion of the collected droplet is negligible, and only the effect of the latter on the former need be considered. Thus $\mathbf{u}_1$ is negligibly small, $\mathbf{V}_2$ remains constant, and the equations of motion become,

$$\left.\begin{aligned} &\mathbf{u}_2 = V_2\,\mathbf{F}_2\,(\mathbf{V}_2), && \mathbf{u}_1 = 0 \\ &\mathbf{R}_1 = 2\,(\mathbf{V}_1 - \mathbf{u}_2)\,a_1/\nu, && \mathbf{R}_2 = \mathbf{R}_2^\circ \\ &\dot{\mathbf{V}}_1 = \frac{6\pi\nu^2\rho}{m_1}\,(\mathbf{R}_1^\circ\, S\,(R_1^\circ) - \mathbf{R}_1\, S\,(R_1)), && \dot{\mathbf{s}}_1 = \mathbf{V}_1 - \mathbf{V}_2 \end{aligned}\right\} \quad . \quad . \quad (7)$$

where $\mathbf{s}_1$ describes the trajectory of the collector with respect to the collected.

The collector droplet tends to acquire the velocity of the fluid within the wake in addition to its own terminal velocity, and the trajectory describes a curved path toward the collected droplet.

As R_2 increases, forces tending to draw droplets together also rapidly increase. The closer R_1 is to R_2 the smaller is the velocity of approach and the greater is the effect of the wake upon the trajectory of the larger droplet, and the smaller the effect of its inertia due to the greater length of time during which the collector droplet lies within the wake. This is illustrated by the trajectories of Fig. 3.

The relative velocity of approach increases as the separation decreases. However the total change in velocity experienced by the collector is only of the order 20 per cent of its own terminal velocity. This may be seen from Figs. 2 and 3.

At close approach, interaction effects become appreciable and must be allowed for in the computations, that is, when the effect of u_1 becomes appreciable, of the order 10-15 per cent of V_1. Reference to Fig. 2 shows that this occurs only when the larger droplet lies toward the rear of the smaller at a separation of about 5 a_1 or less. The velocity of the smaller droplet remains unchanged until this separation is attained. This agrees with the observations of Telford, Thorndike and Bowen (1955) as illustrated by their Fig. 4.

In cases of near miss and at grazing contact the larger droplet emerges from the wake at a distance of some radius units to the rear as may be seen from the trajectories illustrated in Figs. 2 and 3. As the droplet emerges from the wake it passes through a region of very small flow velocity caused by the smaller droplet, into the region where both $\mathbf{u}_1$ and $\mathbf{u}_2$ are directed mainly radially outward, varying with separation r, as r^{-2}, which cause mutual repulsive forces to behave in a manner similar to electrical forces of a Coulomb type.

Fig. 2 shows the effect of these repulsive forces on the trajectories. As the collector passes the smaller droplet it tends to lose the momentum it acquired from the wake, so that pairs of droplets of very nearly equal size and terminal velocity continue to fall at very nearly the same rate and repel one another (cf. trajectory A of Fig. 2). As the difference in the relative terminal velocities increases the collector passes the smaller more rapidly and the degree of separation caused by repulsion diminishes (cf. trajectories B and C).

For any fixed difference in terminal velocities an increase of R_2° causes the width the wake to decrease, and the larger droplet to emerge from the wake at a greater istance, in radius units, to the rear. The repulsive forces which follow have correspon-ngly greater time to take effect. The collision cross-section must therefore finally ecrease as R_2° increases.

For any particular pair of droplets there will be at least one trajectory which allows azing contact. Droplets falling between this trajectory and the axis of motion of the naller droplet will collide and coalesce, those outside will miss. The equations of motion ere solved numerically using the previously computed flow patterns, for $R_1^\circ = 1$, 4, 0 and 40. The integration commenced with the collector droplet at a distance 100 a_1 the rear of the smaller droplet, and at some chosen distance from its axis of motion. he trajectory given by s_1 was traced by step by step integration, the quantity F being btained at each step from curves such as in Figs. 2 and 3.

Mutual interactions at close approach were accounted for to sufficient accuracy by pplying the additional drag force experienced by the smaller droplet, to the larger, ultiplied by the mass ratio, which differed little from unity, with reversed sign. This ad the effect of allowing the smaller droplet to be regarded as fixed. This corresponded osely to allowing the flow $\mathbf{u}_2 - \mathbf{u}_1$ to act upon the larger droplet.

In each case a search was made for the trajectory which allowed only grazing contact y varying the distance from the axis of the smaller droplet at the starting point. The nal estimate for this trajectory was made by interpolation between computed trajectories.

The effective collision cross-section thus obtained depends upon the starting point the integration since the trajectories are uniformly curved and depart further from ie axis of motion for progressively distant starting points. The cross-section tends to finite limit, with increasing initial separations, and for any fixed difference $R_1^\circ - R_2^\circ$; nd increases steadily as the droplets approach equality of size where it becomes inde-nitely large.

The cross-section at infinite separation was obtained by extrapolation of the chosen ajectory backward to infinity from the starting point, by fitting to it a solution of the quation of motion valid at large separations. At large distances $\mathbf{u}_2$ is sufficiently small ompared with $\mathbf{V}_2$ for the effect upon the drag force of variations of the rate of fall to be eglected, so that the equation of motion for the larger droplet becomes

$$\dot{\mathbf{V}}_1 + \lambda_1 \mathbf{V}_1 = \lambda_1 (\mathbf{V}_1^\circ + \mathbf{u}_2) \quad . \quad . \quad . \quad (8.1)$$

here

$$\left.\begin{aligned} \mathbf{u}_2 &= C_2 e^{-R_2^\circ y^2/8x} (\mathbf{x} + 2\,\mathbf{y})/x^2 \\ \lambda_1 &= \frac{9\nu\rho}{2a_1^2\sigma} S(R_1^\circ), \quad C_2 = \tfrac{1}{2} V_2^* R_2^\circ A_0(R_2^\circ), \end{aligned}\right\} \quad . \quad . \quad (8.2)$$

nd σ is the density of the droplet. The relative velocity $\mathbf{v}\,(= \mathbf{V}_1 - \mathbf{V}_2^\circ)$ satisfies the quation,

$$\dot{\mathbf{v}} + \lambda_1 \mathbf{v} = \lambda_1 (\mathbf{v}^\circ + \mathbf{u}_2) \quad . \quad . \quad . \quad . \quad (9)$$

here

$$\mathbf{v}^\circ = \mathbf{V}_1^\circ - \mathbf{V}_2^\circ$$

If accelerations are small, as will be the case if $\lambda_1^{-1}\,\mathbf{u}_2$ is small compared with unity, may be neglected and the trajectory corresponds to a solution of

$$\frac{d\eta}{d\omega} = -\frac{\eta}{\omega + e^{\eta\omega}} \quad . \quad . \quad . \quad . \quad (9.1)$$

here

$$\left.\begin{aligned} \omega &= C_2/v^\circ x \\ \eta &= R_2^\circ v^\circ y^2/8C_2 \end{aligned}\right\} \quad . \quad . \quad . \quad (9.2)$$

Eq. (9.1) was solved numerically for positive ω, starting at $\omega = 0$ (or $x = \infty$) for various values of η_0, the starting value of η. At $x = 100$, the starting point of the main trajectory, C_2, v_0, and R_2° were known, so that ω and η could be obtained by interpolation and a value of y at $x = \infty$ obtained.

Approximate values of the starting velocities for the trajectory calculations at $x = 100$ were obtained for sufficiently large λ_1 and x from eq. (9), thus

$$\mathbf{v} = \mathbf{v}^\circ + \mathbf{u}_2 - \dot{\mathbf{v}}/\lambda_1$$
$$= \mathbf{v}^\circ + \mathbf{u}_2 - \lambda_1^{-1} (\mathbf{v}^\circ + \mathbf{u}_2 \cdot \nabla) \mathbf{u}_2 + 0 (\lambda_1^{-2} u_2^3)$$

Since at any starting point, $\mathbf{u}_2$, $\mathbf{v}^\circ$ and λ were all known, an approximate initial $\mathbf{v}$ was obtainable. In practice it was found that the trajectory was inappreciably affected by slight errors in this estimation of $\mathbf{v}$; the droplet rapidly settled down to the motion demanded by the equation of motion, (7).

Fig. 1, which gives the linear collision cross-sections (σ/a_1) for droplets of different sizes as a function of the radius a_1 and the ratio $a_1 - a_2/a_2$, clearly shows the effects mentioned. This figure has been plotted from a number of trajectories computed as though the droplets approached from infinite separation. For any fixed a_1, as equality of size is approached, the cross-section steadily increases, as it does also for increasing a_1 and constant $a_1 - a_2/a_2$.

With increasing R_1° the wake narrows, and the collision cross-section must, at some stage, reach a maximum and thereafter decrease. From Fig. 1 it would seem that this maximum, although outside the range of computation, is likely to lie in the neighbourhood of $R_1^\circ = 100$, or $a_1 = 300\ \mu$.

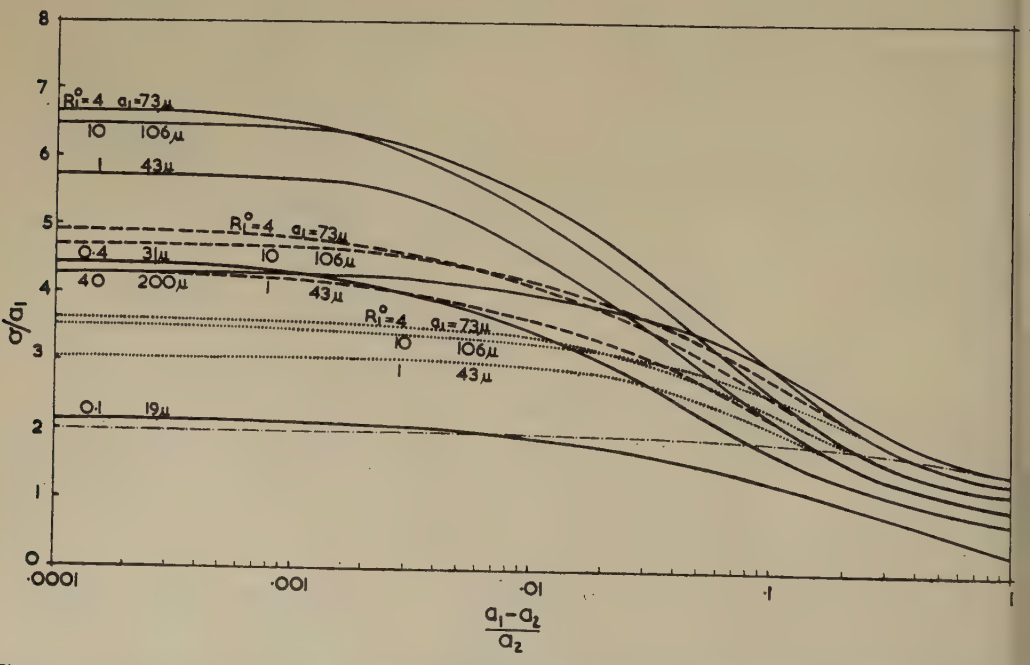


Figure 4. Linear collision cross-sections. The figure shows collision cross-sections between droplets of nearly equal sizes, similar to those of Fig. 1, for different distances of approach. The full lines correspond to initial separations of 100 a_1, the hatched lines to separations of 50 a_1, and the dotted lines to separations of 25 a_1. The geometrical cross-section is shown by the remaining line.

The curves show the effect of the wake to be most pronounced for droplets of R° about 4, or a_1 about 70-80 μ.

Fig. 4 shows the cross-sections referred to definite distances along the wake, namely 00 a_1, 50 a_1, 25a_1. From these the effect is seen of a narrowing wake where, between droplets of identical size, the cross-sections reach a maximum with increasing a_1 or $R_1°$. This occurs for droplets of about 70 μ radius, or $R°$ about 4.

Because of small-scale turbulence in the atmosphere, droplets will come under the influence of one another at finite separations, and in practice, droplets of about 70 μ adius may possess the highest effective collision cross-sections. The experimental results quoted by Telford *et al.* for the collection efficiency E ($= \sigma^2/a_1^2$) may be compared with Fig. 1. Their value of 12·6 for droplets of radius about 80 μ, assumed to be due to direct collisions between droplets of nearly equal size only, may be accounted for on the basis of Fig. 1 if the average difference in radius between droplets is only 6·5 per cent. Alternatively, assuming the distance of approach to be controlled by residual turbulence, and allowing for their droplet distribution, the figure is accounted for if the mean distance of approach is about 30 a_1 or 2·5 mm.

(*b*) *Indirect collision*

As the larger droplet passes in close proximity to the smaller it may be possible for the latter to enter the wake of the former. If this happens collection can only occur if it enters the wake within a region in which the flow is at least as large as the relative terminal velocity, i.e. $u_1 > (V_1° - V_2°)$, in order that the smaller droplet may accelerate downward toward the larger and so collide. In cases in which direct contact is just avoided, and relative terminal velocities are sufficiently small, the larger droplet emerges from the wake of the smaller with a velocity nearly equal to the sum of its terminal velocity and the relatively small velocity it recently experienced within the wake. This additional velocity is damped out by drag forces as the collector overtakes the smaller, and repulsive forces become effective. The droplets fall at almost the same speed and spread apart (cf. trajectory A of Fig. 2).

Droplets of closely similar size separate to such a degree that neither droplet enters the wake of the other, or does so at such great separations that collision cannot occur.

With increasing size difference, droplets pass one another more quickly. The degree of separation suffered becomes less marked since there is less time for the repulsion to take effect (cf. tracks B and C of Fig. 2). In such a case $v°$ is so large, and the region for which $u_2 > v°$ is so small, that the smaller droplet, although little deflected by repulsive forces, does not enter the region of the wake which would allow collision to take place.

Further, for small droplets, $R < 1$, the wake is relatively weak, for larger drops, $R \geqslant 10$, the wake although intense, is narrow. Computation of various trajectories in the first case showed that repulsive effects failed to allow the smaller droplets to enter the wake at any significant point, and indirect collision could not occur. In the latter case the wake was so narrow that, even with its greater intensity, droplets of relatively large difference in terminal velocity, and relatively little deflected by repulsion, did not succeed in entering the wake at a significant point, and again no indirect collision could occur.

However, in the range between $R = 1$ and 10, it seems likely that indirect collisions may occur occasionally. Cases were found in the computations related to $R = 4$ in which indirect collision could take place. In this region of size the repulsive forces cause droplets for small $v°$ to suffer large separations, and entry into the wake occurs too far back for collision to occur. For large enough $v°$ the separation by repulsion although small, does not allow entry into the significant region of the wake. Between these limiting situations the wake is sufficiently intense and wide for the smaller droplet to enter and be accelerated sufficiently to cause collision. It was found that droplets travelling within

a small range of trajectories lying outside the trajectory of direct grazing contact could suffer this type of indirect collision. No detailed study of this type of trajectory was made and the effects of such collisions on the collection efficiencies have not been taken into account in the presentation of the results shown in Fig. 1. Fig. 5 illustrates a typical group of trajectories which were computed showing indirect collision. The tracks are of the path of the larger droplet relative to the smaller placed with its centre at the origin.

The Reynolds numbers of the droplets lie close to 4·0 and differ by about 10 per cent only. Track ' a ' of Fig. 5 shows a case in which the wake is sufficiently strong to reverse the relative velocities, but not to overcome repulsive forces, and the droplets eventually separate. In track ' b ' the tendency for repulsion is less marked, but separation again occurs. In the case ' c ' the wake, within which the droplet is retained, is entered well to the rear, repulsive forces are not experienced and collision occurs. In ' d ' the droplet enters a region where u_2 is approximately equal to $v°$ and hovers before returning to collide. In case ' e ' the wake is entered too far back for collision to occur. The marks on the tracks indicate equal time intervals.

The effect illustrated in Fig. 5 was found not to occur for droplets of closely equal size, i.e. $a_1 = 70\ \mu$, $(a_1 - a_2)/a_2 = \cdot 0075$, or for droplets of size difference greater than $(a_1 - a_2)/a_2 = \cdot 075$, but was found for an intermediate difference in size, $(a_1 - a_2)/a_2 = \cdot 032$, to which Fig. 5 refers.

Thus for droplets of about 70 μ in radius indirect collision may occur when the larger droplet approaches within an annulus in any plane perpendicular to the direction of fall and surrounding the disk of direct collision. Droplets of this size may experience effective collision cross-sections considerably greater than those illustrated in Figs. 1 and 4.

The computation of trajectories such as those of Fig. 5 proceeded in a manner similar to those in cases of direct collision.

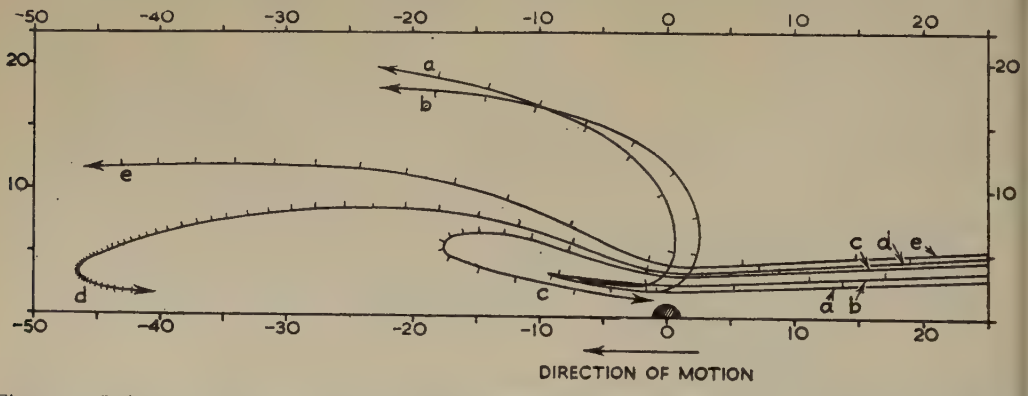


Figure 5. Indirect collision. Trajectories showing indirect collision due to the smaller droplet entering the wake of the larger. In cases ' a ' and ' b ' repulsive forces predominate; in ' c ' and ' d ' indirect collision occurs; and in case ' e ' the wake is entered in a region too weak to cause re-approach and collision. The trajectories correspond to $a_1 = 73\ \mu$, $(a_1 - a_2)/a_2 = 0{\cdot}044$. The marks on the curves indicate equal intervals of time.

6. Coalescence of droplets of widely differing sizes

As the size of the collector droplet increases in relation to the smaller droplet the relative velocity becomes so large that the deflecting forces have too little time to take appreciable effect and the collision cross-section approximates to the geometrical cross-section of the droplets. This is seen in Fig. 1 for droplets of 100-200 μ radius and $(a_1 - a_2)/a_1$ of the order of unity.

If, however, the collector droplet is sufficiently large, the forces due to fluid flow in the cushion of air ahead of it become sufficiently large to overcome the inertia of the smaller droplet and deflect it to one side. Thus the collision cross-section for droplets of widely differing sizes must decrease below the geometrical cross-section.

When this situation applies, the effect of the smaller droplet on the larger is negligible. The mathematical problem is reduced to the study of the trajectories described by the smaller under the influence of the larger. The equations of motion are

$$\left.\begin{aligned} \mathbf{u}_1 &= |\mathbf{V}_1^\circ|\,\mathbf{F}(\mathbf{V}_1^\circ), & \mathbf{u}_2 &= 0 \\ \mathbf{R}_2 &= 2(\mathbf{V}_2 - \mathbf{u}_1)\,a_2/\nu, & \mathbf{R}_1 &= \mathbf{R}_1^\circ \\ \mathbf{V}_2 &= \frac{9\nu^2\rho}{2a_2^2\sigma}\left[\mathbf{R}_2^\circ S(R_2^\circ) - \mathbf{R}_2 S(R_2)\right], & \dot{\mathbf{s}}_2 &= \mathbf{V}_2 - \mathbf{V}_1^\circ \end{aligned}\right\} \quad . \quad . \quad (10)$$

These equations were solved numerically for the various values of R_1° for which $\mathbf{F}$ was known, R_2° being taken sufficiently small for the effect of u_2 on V_1 to be neglected. The technique of solution was similar to that adopted for Eq. (7).

As the velocity of the collector droplet increases, the field of flow at some distance from its forward surface is directed almost entirely radially outward from its centre and Table 1 shows it to increase somewhat more slowly than linearly with R_1°. In the neighbourhood of the forward surface of the droplet the flow changes rapidly. Within a boundary layer of thickness of the order $a_1/R_1^{\frac{1}{2}}$ the flow increases in magnitude and approaches the velocity of the droplet. The effect may be seen in Fig. 2. This change becomes more marked as R increases and for $R = 40$ occurs within a layer about $a_1/6$ thick. A stagnation point lies on the axis of motion and the front surface of the droplet.

The equations of motion, (10), were solved numerically as before, and a search was made for the particular trajectory which allowed only grazing contact by selecting varying distances from the axis of motion of the collector droplet for the starting point at a separation of 100 a_1. The actual integrations were carried out automatically from the starting point, up to separations of about 10 a_1, by use of the analytic approximation (3) for the flow field. From there onward, where the simple analytic representation was inadequate, $\mathbf{F}$ was obtained from diagrams like Fig. 2. At very close approach more detailed diagrams

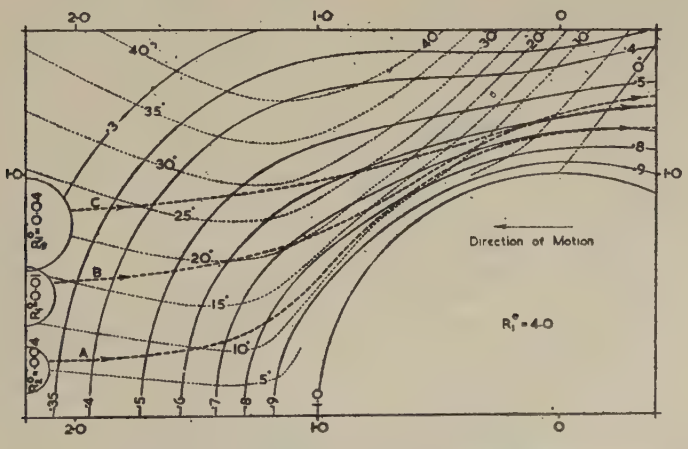


Figure 6. Trajectories of droplets of widely differing sizes. The heavy dotted lines, A, B, and C, show the trajectories of droplets of radius, a_2, equal respectively to 14, 8·8, and 6·5 μ, relative to a freely falling droplet of radius a_1, equal to 73 μ. They show the effect upon the smaller droplet of the boundary-layer type of flow. Other lines show the flow around the larger droplet. The relative sizes and distances from the axis of motion of the larger droplet at infinite separation are also indicated.

like those of Fig. 6 were used. In this manner special computation of starting velocities was avoided, the droplet being allowed to assume its correct velocity during the first stages of integration.

It is again conceivable that collision may occur by direct contact with the forward surface of the collector droplet, and indirectly by being caught up in the wake and making contact with the rearward surface.

(*a*) *Direct collision*

Forces experienced by small droplets due to the flow velocity u_1 are directed radially outward for $R_1^\circ > 1$ up to points relatively close to the surface of the collector. This tends to deflect the lighter droplet directly away. As the smaller droplet enters the boundary-layer type of flow close to the surface of the collector it experiences further strong forces as $\mathbf{u}_1$ approaches $\mathbf{V}_1^\circ$, and is therefore thrust strongly forward and outward resulting in a tendency to move parallel to the surface of the larger droplet. This is illustrated by the trajectories shown in Fig. 6. Collision cross-sections thus depend largely upon the value of $A_0\,(R_1^\circ)$, and upon the thickness and state of development of the boundary layer, and no simple analytical approximation for them seems to be available.

As R_2° is reduced, and thus $(R_1^\circ - R_2^\circ)/R_2^\circ$ increased, for any fixed R_1°, a stage is reached at which no direct collision occurs. The small inertia and large field forces are such as to deflect away all smaller droplets not lying on the axis of motion. Even a small droplet travelling along the axis suffers deceleration to such a degree that it finally comes to rest with respect to the collector where $V_2^\circ + u_1$ is equal to V_1°. This effect was also detected by Langmuir and Blodgett (1945) in their theoretical estimates of collision cross-sections, although the sizes estimated here at which collision cross-section becomes zero differ from theirs. This is probably due to the different fields of flow adopted in our calculations.

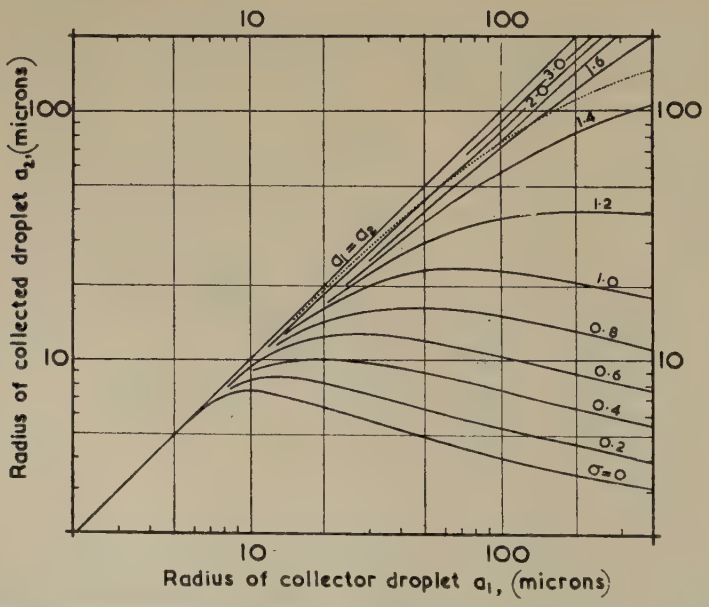


Figure 7. Range of droplet sizes capable of collision. The figure shows contours of constant linear cross-section, σa_1, as a function of both droplet radii. Collision cannot occur for droplets which fall below the $\sigma = 0$ curve, and cases above the straight line $a_1 = a_2$, have no meaning. The dotted line represents geometrical cross-sections.

Fig. 1 indicates how the collision cross-section decreases to zero for any given size collector droplet less than 200 μ in radius. This is shown more clearly by Fig. 7 in hich curves of constant collection efficiency are shown plotted against the radii a_1 and measured in microns. The lowest curve corresponds to zero collection-efficiency d the straight line corresponds to $a_1 = a_2$. In particular it is to be noted that the inimum size of collectable droplets decreases as the size of the collector increases beyond $= 10\ \mu$.

(*b*) *Indirect collisions*

On the basis of the trajectory computations it seems unlikely that indirect collision ay occur by capture of small droplets within the wakes of the large droplets. In all ses, either the smaller droplet possesses too great an inertia and terminal velocity to captured within the wake, or is thrown clear of that part of the wake sufficiently intense cause capture. For R_1° of the order unity, the wake is wide but not sufficiently intense cause capture, whilst for R_1° of the order of 10 or greater, the wake, although intense, too narrow.

7. Coalescence between small droplets

In the case of small droplets, i.e., $R^\circ < 1{\cdot}0$, the fields of flow do not exhibit the esence of the wake to a marked degree, and the boundary layer is completely absent. he fields of flow in the neighbourhood of the droplet differ little from those given by okes, although at sufficiently large distances the field eventually behaves irrotationally. he situation differs from those already considered, in that mutual interactions operate er ranges much larger than either droplet radius.

The computation of the motions is greatly complicated by the importance of mutual teractions. However, the analytical expressions, (4), for the flow fields were used in proximate automatic integration of the equations of motion as in § 3 and some trajectories ere obtained.

The tendency for droplets to fall at their terminal velocities with respect to the local w, which would exist in their absence, allows collision to take place if the droplets e sufficiently close to being the same size. As R_1° decreases, the degree by which droplets ay differ in size, and also collide, decreases.

That collision can take place between droplets of nearly equal size may be seen as llows. If one droplet approaches a second from above, both falling under the influence gravity, each will tend to fall faster, the lower being pushed downward by the forward eld of that behind, which in turn, will be pulled downward by the field of the lower. the two droplets are not falling in the same vertical path both will move sideways in e same direction, i.e., if the lower lies to the right, both will move along a path sloping ward the right, and if to the left the motion will be toward the left. The inward component flow to the rear of the droplet is larger than that at a similar point to the front by an nount of the order of R^2. Thus the upper droplet will tend to accelerate slightly more pidly than the lower and will approach the lower droplet even if both are of identical ze. If the difference in sizes is small enough contact will occur. However, as the sizes crease, the relative velocity of approach becomes small compared with the forward locity, in fact of the order R^2, and the inertias are more readily overcome by deflecting rces and collision avoided.

The computations indicate that the cross-section tends to a limit as both $R_1^\circ - R_2^\circ$ d R_1° tend toward zero, this limit being less than the geometrical cross-section and about 7 times the radius of either droplet. This is shown in Fig. 1 for the cases $a_1 < 10\ \mu$ d $(a_1 - a_2)/a_2 < 0{\cdot}001$. Fig. 7 shows how the range of sizes for which coalescence n take place decreases as the droplet size decreases.

These factors, the large range of mutual deflecting forces and the slowness of approac of two small droplets, together with possible effects upon their approach of microtu bulence of the atmosphere, seem to indicate that collision rates between droplets of size less than about 8-10 μ would be inappreciable in affecting the rate of formation of raindrop

8. Conclusion

It is found, on the basis of hydrodynamical effects alone, that the collection efficienc or collision cross-section, of a droplet of a size greater than 10 μ in radius may be larg for the capture of droplets of sizes little smaller than itself, and may far exceed the geo metrical cross-section, by a factor of 10 to 100. This is due to the suction effect arisin from the fluid flow in the wake of a falling droplet.

As the size of the collected droplet decreases the collision cross-section decrease toward values close to the geometrical cross-section. For very small collected droplet the decrease continues, and the collision cross-section reaches zero for some definit size of droplet, below which no collection can occur. This reduction is due to the sma inertia of the collected droplets and the deflecting and cushioning effect of the flui flow to the front of the collecting droplet.

In the case of droplets of nearly the same size the relative velocities are small and th time taken to complete an approach is correspondingly long. Actual collection rate may therefore be considerably affected by factors such as turbulence which would lim the amount of time during which one droplet may be affected by the wake of anothe Although cross-sections are given as though referred to an infinite distance of approac (Fig. 1), actual collision cross-sections may be more nearly equal to those referred to close distances (Fig. 4).

Enhanced collision cross-sections between droplets of nearly equal sizes are mainl due to direct collisions in which the larger droplet overtakes the smaller, but for droplet in the neighbourhood of 70 μ radius it seems likely that additional collisions may occu by an indirect process in which the larger droplet fails to collide initially and allow the smaller to enter its wake and eventually collide with it. For collecting droplets les than 10 μ radius, collision can occur only between droplets of very nearly the same size This, coupled with the slow speed of approach and possible effects of turbulence, indicate it is likely that collision processes between small droplets of 10 μ or less in radius ar unimportant in the formation of raindrops.

The computations of the trajectories were carried out almost entirely on the C.S.I.R.C computer, without which solution of the equations of motion would have been quit impracticable.

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Seiches in Lough Neagh

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SUMMARY

Records of seiches in Lough Neagh during April 1949 were investigated. Fourier analysis of the records dicated three period bands centred round 1·6 hr, 1·15 hr, and 0·75 hr. The two longer periods appeared have a nodal line across the centre of the lake, parallel to the shorter sides. Methods of deriving these eriods in terms of the shape of the lake basin are discussed.

The amplitudes of the seiches were compared with the wind records and a high degree of correlation as obtained between the amplitude and $Wd\theta/dt$, where W is the wind speed and $d\theta/dt$ is the rate of change f wind direction.

An estimate was made of the coefficient of damping of the seiche amplitude and it was found to correspond a decay to half value in 10 hr.

1. INTRODUCTION

The history of the investigation of seiches is a long one. Merian (1828) studied scillations in long rectangular basins and deduced a formula for the period when the ectangle was long and narrow and the depth constant. Chrystal and Wedderburn (1905) nade a special study of the seiches in Loch Earn and Loch Treig and extended Merian's vork to cases where the simple conditions were not obtained. Sternbeck (1915) and Defant (1918) have also extended this work to bays and gulfs. More recent work on eiches has been carried out by Bottomley (1956) on Lake Wakatipa in New Zealand nd Zorè (1955) on the Bay of Kastela in the Adriatic Sea.

Most of this work has been concerned with determining the seiche period in terms f the dimensions of the basin rather than relating the amplitude of the seiche to meteoro- ogical and other factors but Van Dorn (1953) considered an artificial lake of length 40 m, breadth 60 m, and depth 2 m and found that the seiche amplitude was a function f the wind speed when the wind was blowing along the lake.

The present paper describes an investigation of seiches in Lough Neagh in N. Ireland vhere measurements were carried out in April 1949. The investigation was mainly oncerned with the variation of seiche amplitude with meteorological factors, as it was elieved that the results would also be applicable to gulfs and bays. There was the advant- ge at Lough Neagh that the effect of seiches was not masked by tides and external effects uch as edge waves on the continental shelf, as would be the case in the sea. The lake s very suitable for this investigation, as it is about 16 mi long and 9 mi wide, and at Aldergrove, about 4 mi away, continuous records of wind speed and direction are taken y Dines anemometer. The winds were measured at a height of 12 m, and it could be rgued that they were not the most suitable for comparing with seiche amplitude but ney proved satisfactory in the earlier investigation of the tilt of the lake surface (Darbyshire nd Darbyshire 1955). Measurements of the temperature and atmospheric pressure were lso available.

2. LAYOUT OF RECORDING STATIONS AND APPARATUS

The situation of the recording stations and a description of the instruments have ready been given in the previous paper. A map of Lough Neagh is shown in Fig. 1 nd following the notation used in the previous paper, the stations will be indicated by umbers, (1) for Toome Bridge, (2) for Newport Trench, (3) for Ellis Cut, and (4) for

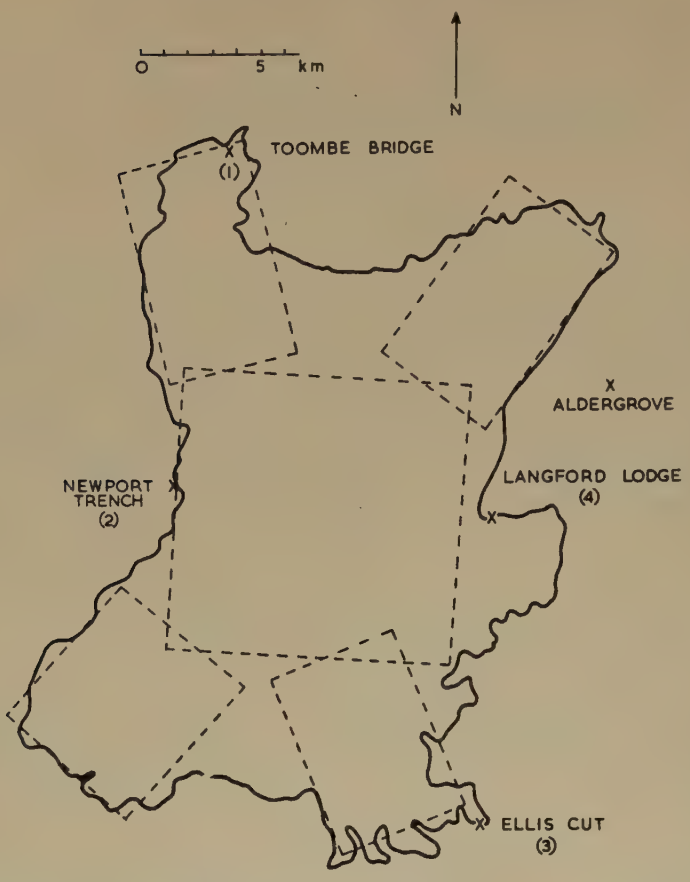

Figure 1. Map of Lough Neagh.

Langford Lodge. The water-level recorders consist of a still well so constructed that waves of period less than 15 min were not transmitted, and the up-and-down motion of the float was recorded, reduced by a factor of three, on the chart of a pen recorder. Examples of water-level records are shown in Fig. 5. The records were taken continuously from 10 April to 26 April 1949.

3. Analysis of periods

The records show, in addition to seiches, the effect of the tilt of the lake surface due to the wind stress, investigated in the previous paper. It was desirable to eliminate this effect and this was done by subtracting from the observed water levels the smoothed values obtained by meaning the observed values over three hourly intervals. Values of water level derived in this manner were plotted at six minute intervals and the outline blacked in so that the record was suitable for analysis with the analyser described by Barber, Ursell, Darbyshire and Tucker (1946). The seiche records showed the greatest activity from 16 to 18 April and from 21 to 22 April, and the four records for both these intervals were analysed. The analyses are shown in Fig. 2. The spectra show bands centred about 1·6 hr, 1·15 hr, and 0·75 hr. The 1·6 hr period is very small on (4), the 1·15 hr period is very small on (2), and the 0·75 hr period is very small on (3). The phase differences corresponding to the periods can be estimated as described by Darbyshire

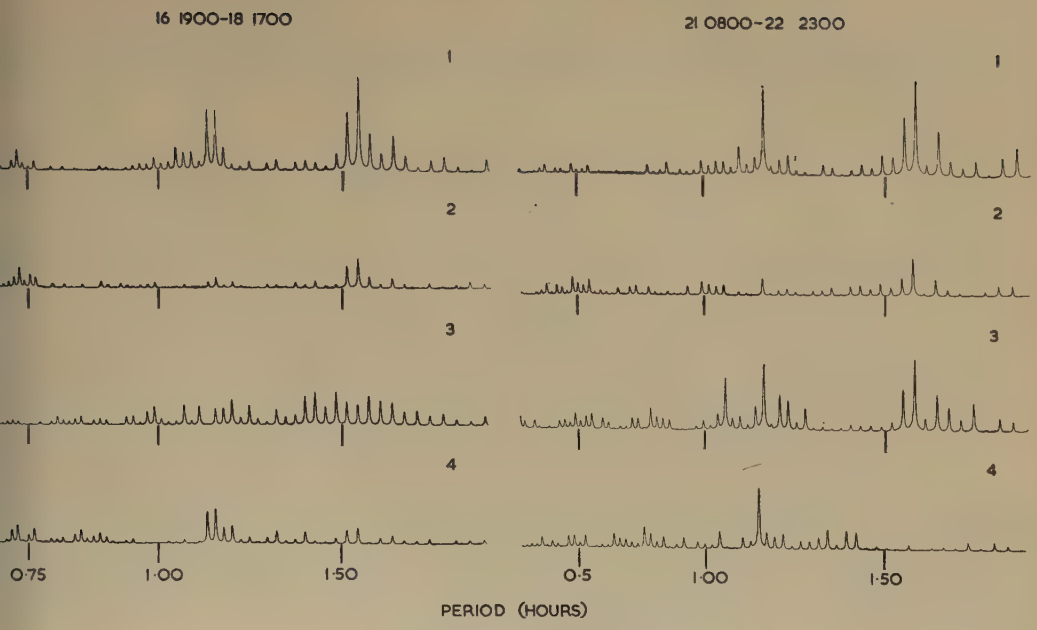

Figure 2. Spectra of Lough Neagh seiches.

nd Darbyshire (1956). The two records under investigation are analysed separately, hen together to give a sum of the two, and then one is shifted with respect to the other by a definite amount, which was taken in this case to be $\frac{3}{8}$ hr, and another analysis made. The phase difference for each period is deduced from the four values obtained for the mplitude of each period. The angles obtained are given in Table 1 (a) for the two sets f analyses. In view of the probable errors involved in these determinations, the values btained are not in disagreement with the values given in Table 1 (b), which are consistent with a system of stationary waves. The ratios of the amplitudes corresponding

TABLE 1 (a). Observed phase difference in degrees

Stations \ Periods	1·6 hr	1·15 hr	0·75 hr
1 and 2	35, 30		157, 180
1 and 3	180, 197	217, 222	
1 and 4		200, 180	0, 53
2 and 3	140, 207		240, 215
2 and 4			227, 231
3 and 4		0, 0	

TABLE 1 (b). Phase difference in degrees for stationary wave system

Stations \ Periods	1·6 hr	1·15 hr	0·75 hr
1 and 2	0		180
1 and 3	180	180	
1 and 4		180	0
2 and 3	180		180
2 and 4			180
3 and 4		0	

to the three periods at each station were found by taking the square root of the sum of the squares of the harmonic components centred round each band. The ratios are given for both times in Table 2.

TABLE 2. RATIOS OF AMPLITUDES

Stations \ Periods	1·6 hr	1·15 hr	0·75 hr
2/1	0·18, 0·19		0·63, 0·98
3/1	0·49, 0·75	0·46, 1·00	
4/1		0·31, 0·36	0·53, 0·87

These results and the low value of the 1·6 hr component for (4) and the 1·15 hr component for (2) suggest that the nodal lines for the two longer-period oscillations are placed somewhere half-way along the lake, not far from the line joining (2) to (4). The amplitude of the 0·75 hr component is too small for any definite conclusions to be made; it may be a higher harmonic of the 1·6 hr period, or it could be a transverse oscillation parallel to the shorter sides of the lake.

4. THEORETICAL DETERMINATION OF THE PERIODS

Merian's formula for a long narrow lake of rectangular shape and constant depth is

$$T = 2l/(gh)^{\frac{1}{2}}$$

where l is the length, and h the depth. Merian's formula is a particular case of the more general formula for the period of standing oscillations in a rectangular tank which is

$$T = 2\,(gh)^{-\frac{1}{2}}\,(m^2/a^2 + n^2/b^2)^{-\frac{1}{2}}$$

where a is the length, b the breadth, and m and n can take integer values from 0 to ∞. If Lough Neagh is taken as a rectangular tank of length 13 mi, breadth 8·7 mi and with a mean depth of 35 ft, then

$$T = 1{\cdot}12 \text{ hr for } m = 1,\ n = 0$$

and

$$T = 0{\cdot}75 \text{ hr for } m = 1,\ n = 1$$

These values are in agreement with the lower values of the observed periods, but the 1·6 hr value is not given.

If Merian's formula is applied along the two 'diagonals' of the lake, from NW to SE and from NE to SW, taking the length of the diagonal as l and the mean depth along it as h, values of 1·6 hr can be found for both cases. Chrystal and Wedderburn (1905) have, however, extended Merian's formula to cases where the basin is not exactly rectangular and the depth is not constant. The motion can be represented by a differential equation

$$d^2V/dx^2 = -\,V/D_x\,(n^2/g)$$

where V is the volume of water which has crossed a given transverse section up to a time t, x is the surface area from one end of the lake to the section taken, D_x is the product of the length of the line drawn from the end of the lake to the section and the cross-sectional

rea, and n is the frequency of the seiche. An approximate solution to this equation can e found by applying Rayleigh's principle so that

$$n^2/g = \frac{\int_0^A (dV/dx)^2\, dx}{\int_0^A (V^2/D_x)\, dx}$$

here A is the area of the lake. The values of V and D_x were worked out from the Admiralty hart of the lake, taking the length along the diagonals, but, as the periods came to 1·1 hr, t appeared that the method was not very suitable for a lake of this shape.

Another way of tackling this problem is by a method, used by G. Neumann (1944), ased on analogy with electrical and acoustical circuits, making use of the impedance of a ake or basin. A narrow canal with one open end has an impedance

$$Z = -i\rho c/bh \cdot \cot nl/c$$

here l is the length, b the breadth, h the depth, and $c = (gh)^{\frac{1}{2}}$. A similar canal with wo open ends has an impedance

$$Z = i\rho c/bh \cdot \tan nl/c$$

The resonance frequency is determined by putting $Z = 0$ and solving for n. The two ormulae give resonance frequencies

$$n = \pi (gh)^{\frac{1}{2}}/2l \text{ and } n = \pi (gh)^{\frac{1}{2}}/l$$

n agreement with the results found by other methods. Neumann extended the method o complex basins which can be split up into rectangular components. One case dealt vith is that of a long canal with two branches at one end, the impedance being :

$$Z = Z_1 + Z_2 Z_3/(Z_2 + Z_3)$$

The suffix 1 applying to the long canal and 2 and 3 to the branches which are regarded s two impedances in parallel. If all the canals are closed at one end

$$Z_1 \text{ (etc.)} = -i\rho c/b_1 h_1 \cdot \cot nl_1/c_1, \text{ (etc.)}$$

Equating to zero, one obtains after some manipulation

$$b_1 c_1 \tan \frac{nl_1}{c_1} + b_2 c_2 \tan \frac{nl_2}{c_2} + b_3 c_3 \tan \frac{nl_3}{c_3} = 0,$$

result previously obtained by Zeilon (1913).

It was decided to try a similar method with Lough Neagh, splitting the lake up into ive rectangles as shown by the broken lines in Fig. 1. The combined impedance Z will hen be given by

$$Z = Z_1 + Z_2 Z_3/(Z_2 + Z_3) + Z_4 Z_5/(Z_4 + Z_5)$$

here

$$Z_1 = i\rho c/b_1 h_1 \cdot \tan nl_1/c_1,$$

nd

$$Z_{2,3,4,5,} = -i\rho c/b_2 h_2 \cdot \cot nl_2/c_2, \text{ etc.}$$

n the calculation bh was taken as the mean cross-section in each branch. It is possible o locate the roots of the equation $Z = 0$, and they are given by values of n corresponding o $T = 1{\cdot}67$ hr and $T = 0{\cdot}78$ hr. At $T = 1{\cdot}15$ hr, $Z = \pm \infty$, so the 1·15 hr period which

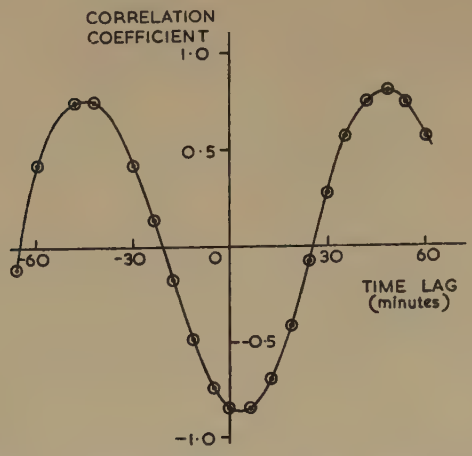

Figure 3. Example of a cross-correlogram of Toome Bridge and Ellis Cut records.

is given by the rectangular approximation would not be obtained by this approximation. The two approximations emphasize two different aspects of the shape of the lake. The more complicated representation gives weight to the elongated corners but not enough weight to the central part of the lake. The central part is adequately represented by the rectangular approximation but this cuts off the corners. The two approximations between them give all the observed periods and it is likely, therefore, that these periods could be deduced rigorously from a more exact representation of the lake basin. This might be possible by using relaxation methods or by making a model of the lake as was done by Bottomley (1956) for Lake Wakatipa.

5. Analysis of amplitudes

It was necessary to get a reliable estimate of the amplitude of the seiche across the lake, as each recorder might be affected by local oscillations of that part of the lake as well as by the main oscillation. The amplitudes were always greater at stations (1) and (3), so that only the records for these two stations were considered. The two sets of records were cross-correlated at different time intervals. Ten hours of record (1) from $t = 0$ to $t = 10$ hr were correlated with different ten-hour lengths of record (3), starting at times ranging from $t = -2$ hr to $t = +2$ hr. Ten hours of (1) and the corresponding fourteen hours of (3) were selected from the records at frequent intervals, and, during the days of greatest activity, 16 to 18, and 21 to 22, the central time $t = 5$ hr was taken every 6 hr. The correlogram for each 10 hr series was plotted as shown in Fig. 3. The covariances corresponding to the maximum positive correlation and the maximum negative correlation were added together, ignoring the signs, and the square root of this sum was taken as an estimate of the seiche amplitude. This term will be denoted by s and its relation to the amplitudes can be shown as follows.

As the 0·75 hr period oscillation is very small, the oscillations are nearly biperiodic so that the displacement of (1) at time t is :

$$x_1 = a_1 \sin mt + b_1 \sin nt + \phi(t)$$

and the displacement of (3) at time $t + t_0$ is

$$x_3 = a_3 \sin m(t + t_0 + \alpha) + b_3 \sin n(t + t_0 + \beta) + \psi(t)$$

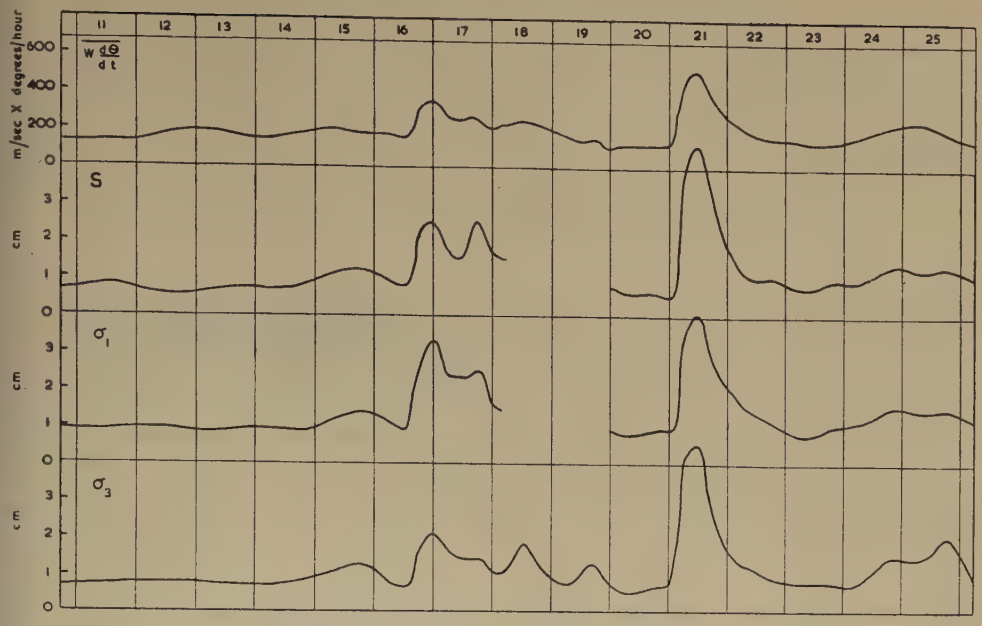

Figure 4. Plot against time of s, σ_1 and σ_2.

$\phi(t)$ and $\psi(t)$ are uncorrelated with each other and with $\sin mt$ and $\sin nt$ so that

$$\int_0^\infty \phi(t) \cdot \psi(t)\, dt = 0, \quad \int_0^\infty \phi(t) \sin mt\, dt = 0 \quad \text{etc.}$$

Now
$$1/T \int_0^T x_1 x_3\, dt = \tfrac{1}{2} a_1 a_3 \sin m(\alpha + t_0) + \tfrac{1}{2} b_1 b_3 \sin n(\beta + t_0)$$

when
$$T \gg 2\pi/m,\ 2\pi/n$$

and provided t_0 is varied over a wide enough range, the sum, neglecting signs, of the maximum positive value and the maximum negative value is :

$$a_1 a_3 + b_1 b_3$$

so that
$$s = (a_1 a_3 + b_1 b_3)^{\frac{1}{2}}.$$

s and σ_1 and σ_2, the corresponding standard deviations for records (1) and (3) are plotted against time in Fig. 4.

6. Comparison with meteorological conditions

Comparison of s with wind speed, atmospheric pressure, and temperature changes did not lead to any useful result. It was considered significant, however, that the two occurrences of greatest activity followed very abrupt changes in wind speed and direction. This is shown in Fig. 5 where records of wind speed and direction, taken from the Dines anemometer, are compared with the corresponding seiche records, the horizontal time scales being made the same for both sets. The times shown are from 16 April, 1700 to 17 0700 and 21 April, 0600 to 1900, (A) indicates Toome Bridge records, (B) Ellis Cut record, (C) record of wind speed, (D) record of wind direction. The abrupt changes in wind conditions are followed in each case by a marked increase in seiche amplitude. These abrupt changes in wind conditions were due to the passage of a meteorological front. The fronts in each case were moving at a speed approximately equal to that of a long wave in

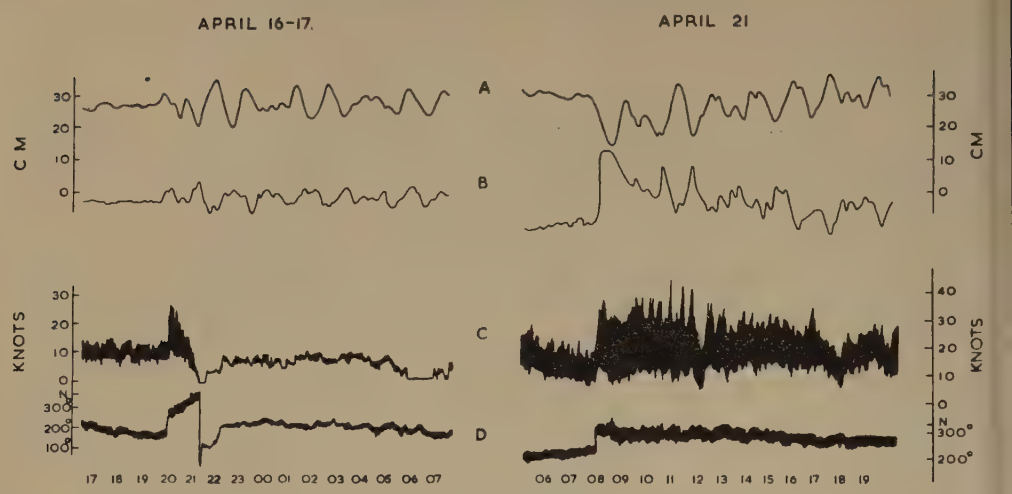

Figure 5. Simultaneous records of seiches and wind speed and direction. (A) Toome Bridge record (B) Ellis Cut record, (C) Record of wind speed, (D) Record of wind direction.

the lake which was 20 kt. It was unlikely, though, that the increase could be due solely to frontal movement, as several other fronts moving with about the same speed passed over the lake during the period investigated with much smaller effect, and only the fronts which involved an abrupt change of wind direction were effective. The variation of wind speed and direction with time seemed then to be the direct factor involved. This would be similar to the effect previously found in the North Sea (Darbyshire and Darbyshire 1956) where the rate of change with time of the pressure gradient across the northern entrance of the sea was found to have an important effect. Attempts to correlate s with the variation with time of the components of the wind speed along and at right angles to the line joining (1) and (3) were not successful but the term $Wd\theta/dt$, where W is the wind speed and θ, the wind direction was more promising. This term was investigated further by measuring W and θ every half-hour over the 10 hr interval to find the root mean square of $Wd\theta/dt$, $\overline{Wd\theta/dt}$. When $\overline{Wd\theta/dt}$ was correlated with s, a correlation coefficient of 0·87 was found with 31 pairs of values. This analysis did not take into account the finite time taken by the seiches to damp out. This was now considered by adding to $\overline{Wd\theta/dt}^2$, a portion of the $\overline{Wd\theta/dt}^2$ evaluated for the 10 hr centred round a time six hours earlier and so on, obtaining finally

$$\underset{t=0\,\mathrm{hr}}{\overline{Wd\theta/dt}^2} + k\,\underset{t=-6\,\mathrm{hr}}{\overline{Wd\theta/dt}^2} + k^2\,\underset{t=-12\,\mathrm{hr}}{\overline{Wd\theta/dt}^2} + k^3\,\underset{t=-18\,\mathrm{hr}}{\overline{Wd\theta/dt}^2} \text{ etc.}$$

the series being continued until the terms become negligible. The correlation coefficients were worked out for various values of k and are shown in Table 3. The maximum

TABLE 3.

k	Correlation coefficient
0·00	0·875
0·25	0·947
0·33	0·959
0·43	0·963
0·50	0·961
0·60	0·948

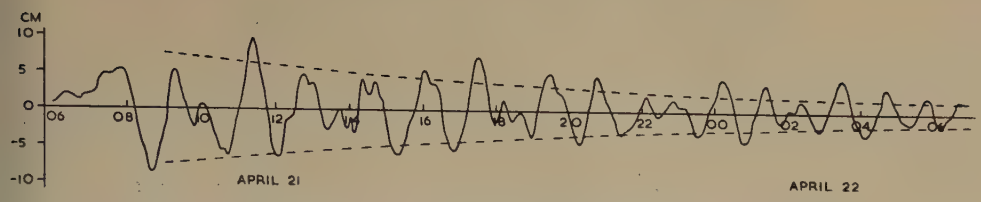

Figure 6. Seiche record for 21 April 1949, with damping curve superimposed.

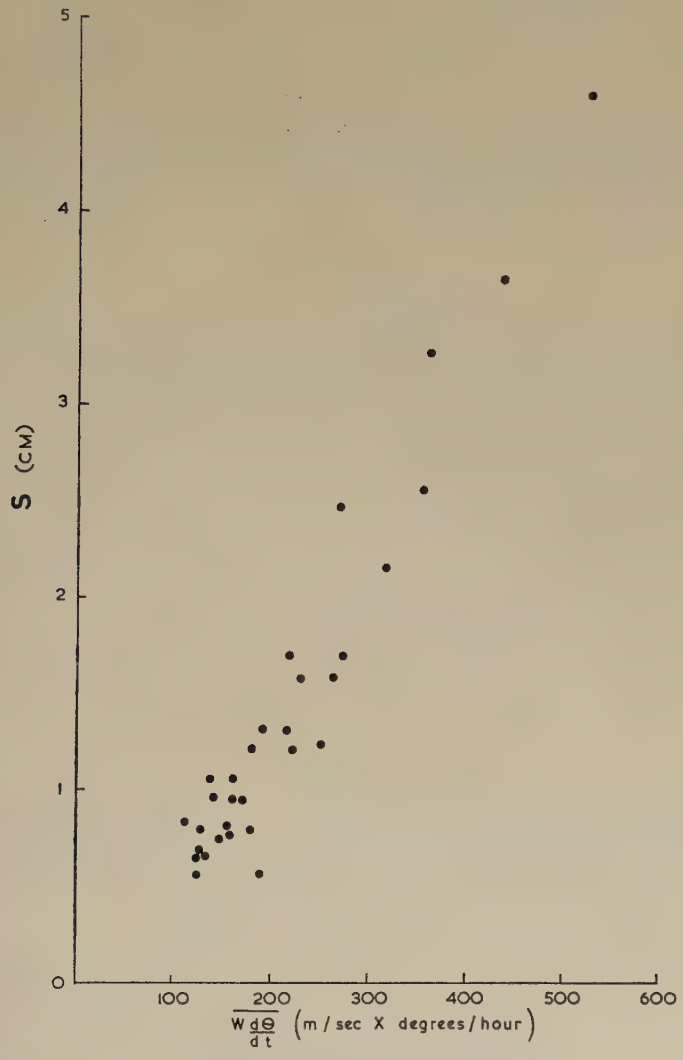

Figure 7. Plot of s against $\overline{Wd\theta/dt}$.

orrelation coefficient, 0·963, was found for $k = 0{\cdot}43$. This in terms of amplitude represents decrease to half the original value in 10 hr. The increase in correlation coefficient from ·87 to 0·96 for 31 pairs of values is statistically significant and although the variation etween intermediate values of k would not pass such a stringent test, the maximum btained is quite pronounced. The optimum value of k can be checked to some extent y reference to the actual records, for the activity on 21 to 22 Apr. was such that an initial hange in $Wd\theta/dt$ was not followed by others of comparable magnitude so that the

amplitude of the oscillation during the succeeding hours could be regarded as being mainly due to the original impulse. The seiche record (after the effect of the tilting of the surface had been removed), is shown in Fig. 6, and the broken line corresponds to a damping to 0·5 in 10 hr. The agreement is not unsatisfactory when it is considered that a certain amount of energy was still being put into the oscillations during this time.

The composite value of $\overline{Wd\theta/dt}$ is plotted in Fig. 4 with s, σ_1 and σ_2; it is also plotted against s in Fig. 7. The regression line does not go through the origin, indicating that a 'threshold' value of $\overline{Wd\theta/dt}$ must be exceeded to initiate seiches.

The significance of these results would be that the lake surface acts analogously to a lightly damped galvanometer; it reaches an equilibrium value when conditions are steady; but when the wind (cf. the voltage) changes suddenly the system starts to oscillate about a new equilibrium value. If the changes are introduced very slowly however, the oscillations about the equilibrium value become smaller and are negligible if the rate of change is made low enough.

7. Conclusions

Analysis of records of seiches in Lough Neagh indicates that there are two main periods of oscillation, 1·6 hr and 1·15 hr, with a nodal line across the middle of the lake parallel to the shorter sides. There is also a smaller 0·75 hr oscillation.

It has not been possible to derive all three periods theoretically from one approximation to the shape of the lake basin, but the three can be derived from two different approximations which emphasize different features of the shape and this suggests that the periods would be derivable from a more exact approximation, involving a great deal of numerical computation.

The amplitude of the seiches was found to be closely related to the product of the wind speed and the rate of change of wind direction.

It was found that the seiches were damped out to half their original amplitude in 10 hr.

Acknowledgments

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Intracloud lightning discharges

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Summary

Slow electric-field changes produced by lightning discharges within clouds have been recorded simultaneously at two stations 13·2 km apart. It is found that the same cloud discharge may give field changes of different types (simple and complex), or of opposite sign (reversal effect), or both, at the two stations. Two main types of complex slow field change are described. The interpretation of slow field changes is discussed in terms of movement of charge. It is shown that, in the most frequent case, the cloud discharge involves the raising of negative charge.

1. Introduction

In this paper we shall describe and discuss observations of slow electric-field changes produced by intracloud lightning discharges. Cloud discharges, for which no channel is visible, are more frequent than discharges to ground. The ratio varies according to location and method of observation; during the present series of electric-field measurements a ratio of 3/1 was found. The two forms of lightning may be distinguished visually, but are more reliably identified by features of the field change. Cloud discharges produce slow, relatively smooth, field changes; discharges to ground have roughly the same total duration, of the order of 0·3 sec; but superimposed on this, at intervals of the order of 0·1 sec, are one or more rapid components with durations of the order of 1 millisec which have been shown, first by Appleton and Chapman (1937), to correspond to the separate strokes to ground. Comprehensive observations by Pierce (1955) have confirmed this identification.

2. Experimental method

The observations of electric field were obtained at two stations recording simultaneously at locations 13·2 km apart, near Orlando in central Florida, during July and August 1955. At each station a field meter (Smith 1954) of the field-mill type is mounted, inverted, 2 m above the ground. The output of each field meter is recorded on a two-channel hot-stylus recorder which uses a heat-sensitive chart, 5 cm wide for each channel, having rectangular coordinates. The frequency response of the system is limited by the recorder; with a step increase of field the final deflection is reached in 10 millisec with an overshoot of 5 per cent. The chart speed of each recorder was alternated, at prearranged 10 min intervals, between a slow speed of 1 mm/sec and a high speed of 10 mm/sec. In this paper we shall discuss the observations recorded at high speed.

The sensitivities of the two channels of each recorder were adjusted to differ by a factor of three, so that the occasional field change giving off-scale deflection on the more sensitive channel would be recorded on the other channel. A selector switch in the field-meter amplifier provides four steps of three in sensitivity. On the most sensitive scale a field of 0·1 volt/cm gives 1 mm deflection on the chart (2·5 volt/cm full scale). The sensitivity was checked after each period of observations by applying a known field to the field meters; the absolute calibration was made during fair weather by comparison

with measurements obtained by the stretched-wire technique, using a radioactive source. Timing marks were put on the records at one-second and one-minute intervals, occasional checks being made against time signals broadcast by radio station WWV. In this way it is easily possible to synchronize the records of the two stations within 0·5 sec. No greater accuracy in timing is necessary since the observations are of close storms with discharges at intervals of about one minute.

The general meteorological situation is found from the records of two weather stations in the area. One of these is the U.S. Weather Bureau station at Orlando, located 2·6 km SW of station 1, and the other is the Air Weather Service station at Pinecastle Air Force Base, 1·4 km S of station 2. Station 2 was 13·2 km due S of station 1. The records are augmented by notes, particularly of the occurrence of rain and thunder, made by the observers at the two field-measuring stations. It had been hoped to locate the discharges approximately by measuring the lightning-thunder interval at the two stations, but it was found that thunder was not noted by the observers following slow field changes. It is assumed that thunder resulting from cloud discharges is relatively weak and is often mistaken for echoing of thunder from an earlier discharge to ground.

The thunderstorms of this area have been extensively investigated by Byers and Braham (1949). During the summer the storms are normally non-frontal and widely spaced and occur with a frequency and regularity which simplifies the observational routine. Thunder is reported at Pinecastle on about 55 per cent of the days in July and August and, on about 85 per cent of those days, thunder is first reported between 1200 and 1800 EST. Simultaneous records at high speed total 351 min for each recording station spread over 11 days of thunderstorm activity. There is, in addition, 203 min of records when only one of the two stations was recording at high speed.

3. Types of slow field change

During this investigation a total of 693 slow field changes was recorded, the two stations being counted separately. Of these, 69 per cent are simple field changes; the remaining 31 per cent are complex, having at least one maximum or minimum during the field change. It is found that the majority of complex slow field changes are almost equally divided between two clearly distinguished forms, which we shall call Type I and Type II. Type I is defined to be a complex slow field change having one turning point (maximum or minimum) and Type II has both a maximum and a minimum. 14 per cent of the slow field changes are complex of Type I and 13 per cent are of Type II; the other four per cent have more than two turning points.

Examples of slow field changes photographed from the original records are shown in Fig. 1. In the Figure, (b) 1, (c) 1 and (d) 1 are Type I field changes; (e) 1 and 2 and (f) 2 are Type II field changes; the remainder are simple field changes.

Further examples of the three types of slow field change may be seen in the records published by Kitagawa, Iizuka, Murai and Kobayashi (1954). These were observed as a thunderstorm passed within several kilometres of the recording station. Pierce (1955), recording at distances of up to 150 km, finds that 15 per cent of the slow field changes are complex. He reports that for the complex slow field changes ' the individual form of most common occurrence ' is what we here call Type II.

Comparison of the records shows that the field change is not always the same at the two stations either in type, as defined above, or in sign of the net change, or both, as may be seen in Fig. 1. We shall now consider the interpretation of these observations, as far as possible without assuming a detailed knowledge of the mechanism of the discharge.

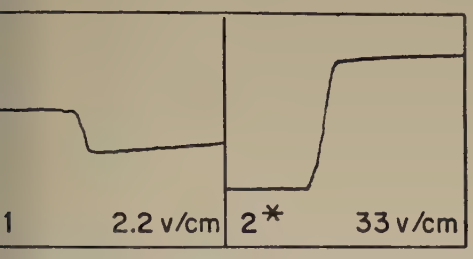
4. The reversal effect

When a cloud discharge destroys a vertical dipole consisting of a charge Q centred a height H and a charge $-Q$ at a height h, the net field change F observed at the ound at a horizontal distance L from the dipole is given by the well-known equation :

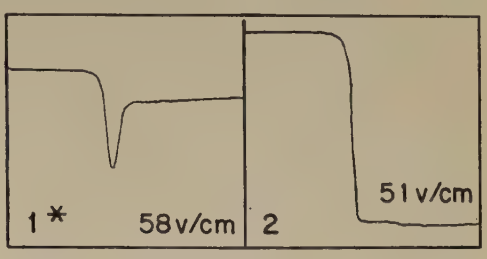
$$F = -90\left[\frac{2QH}{(H^2+L^2)^{\frac{3}{2}}} - \frac{2Qh}{(h^2+L^2)^{\frac{3}{2}}}\right]$$

volt/cm for Q in coulombs and H, h and L in km. There is a certain distance L^*, the versal distance, at which the net change of field is zero, the net change being of opposite gn inside and beyond this distance. It is easily shown that the reversal distance is given by

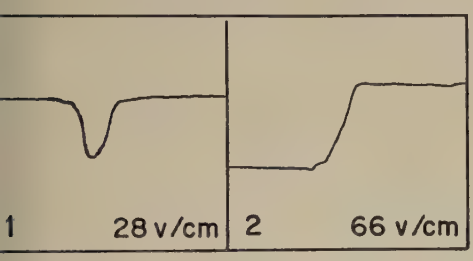
$$L^* = H^{\frac{1}{3}} h^{\frac{1}{3}} (H^{\frac{2}{3}} + h^{\frac{2}{3}})^{\frac{1}{2}}.$$



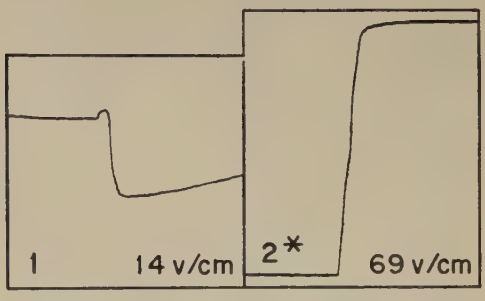
a) 28 JULY 1955: $15^H\ 14^M\ 06^S$



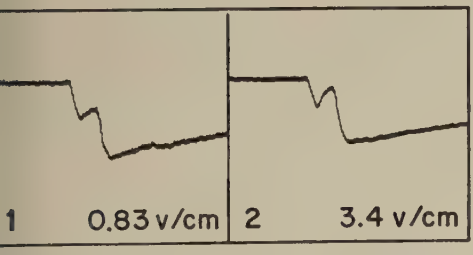
(b) 31 JULY 1955: $17^H\ 36^M\ 47^S$



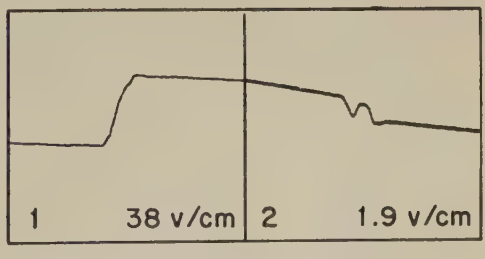
c) 12 AUGUST 1955: $11^H\ 30^M\ 36^S$



(d) 08 AUGUST 1955: $12^H\ 11^M\ 35^S$



e) 30 JULY 1955: $15^H\ 56^M\ 20^S$



(f) 12 AUGUST 1955: $11^H\ 13^M\ 27^S$

1 SEC

gure 1. Slow electric-field changes recorded simultaneously at two stations, 13·2 km apart. The amplitude of each is given. The station known to be closer to the storm is indicated by *.

For purposes of illustration in this and later sections of the paper, we shall consider a vertical positive dipole consisting of a charge of 20 coulomb centred at a height of 7·5 km and a charge of − 20 coulomb at a height of 2·5 km. In this case the reversal distance is computed to be 6·32 km. When, in the more general case, the dipole is inclined the reversal distance depends on the direction from the dipole.

The existence of the reversal effect (Wilson 1920) is well established on a statistical basis from many investigations of the prevailing sign of the net change of field; the present investigation provides the first examples of the effect for individual discharges. There are 48 cases in which slow field changes, including both simple and complex types, were recorded with opposite sign of net change at the two stations. These include 11 cases in which a simple field change was recorded at both stations. In each of the 11 cases the station nearer to the storm centre recorded a positive field change (ranging from 6·1 to 148 volt/cm), while the more distant station recorded a negative field change smaller by a factor of about 10 (ranging from − 0·43 to − 31 volt/cm). In the example shown in Fig. 1 (a) the weather station near station 2 had, four minutes earlier, reported a thunderstorm overhead, which is supported by the report from the weather station near station 1. All the 11 cases indicate the discharge of positive dipoles within the clouds.

For those discharges for which a Type I field change was recorded at one or both stations, 35 cases show the reversal effect, including seven cases in which the net change of field at one station is zero; Figs. 1 (c) and (d) are examples. The reversal effect was also observed for nine cases of Type II field changes. In five of these there was a Type II field change at both stations, with opposite sign of net change, while the other four occurred with a simple field change at the other station; Fig. 1 (f) is an example of the latter.

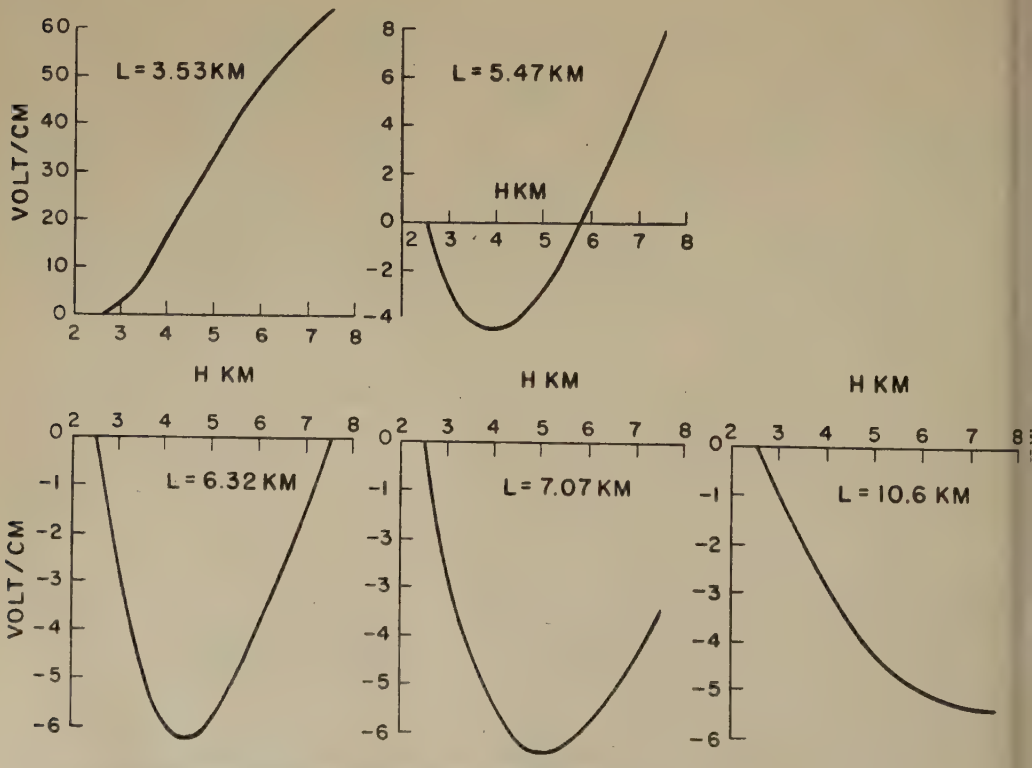


Figure 2. Field changes resulting as a charge of − 20 coulomb is raised vertically from 2·5 km to 7·5 km, showing the variation with distance.

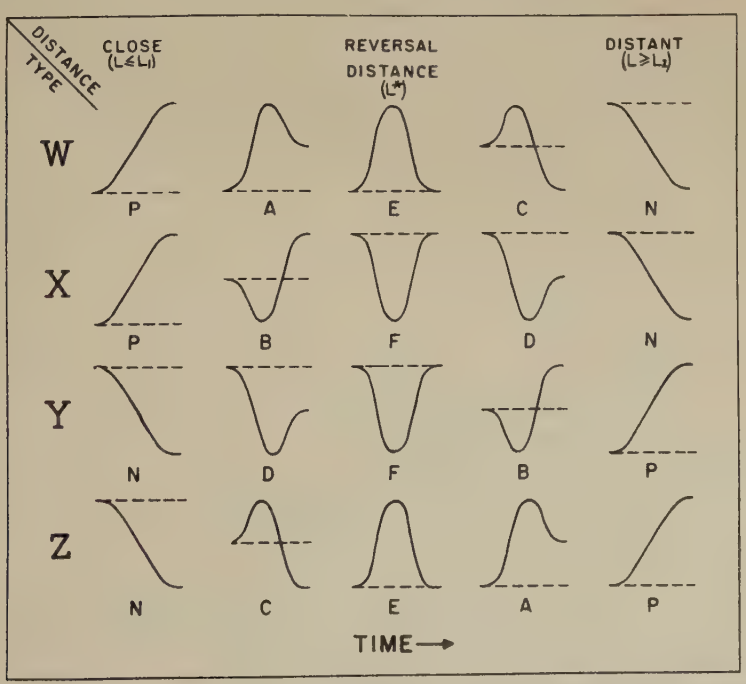
In discussing the reversal effect we have found it necessary to specify nothing about e cloud discharge, except for the initial charge locations. The variation of field during e discharge, which will be discussed in the following sections, can be expected to reveal nething of the manner in which the discharge takes place.

5. Complex slow field changes of Type I

When a Type I field change is recorded at both stations the turning point is always the same sense (maximum or minimum); in seven cases the net change is of opposite gn and in two cases the net change at one station is zero. However, when a Type I ld change is observed at one station the other more frequently records a simple field ange. Of 45 cases of this, 26 show the net change of opposite sign and 19 the net change the same sign.

The occurrence of a turning point in the field change may be interpreted as the move- ent of charge within the cloud. Consider the case of the vertical positive dipole used to ustrate the discussion of Section 4. The field change resulting as the lower negative arge is raised is shown in Fig. 2 for several values of the distance L. There is a minimum the field change for a range of distance between $L_1 = 2\frac{1}{2}h$ and $L_2 = 2\frac{1}{2}H$, which we ll the inner and outer critical distances for the discharge. In the example of Fig. 2, $= 3{\cdot}53$ km and $L_2 = 10{\cdot}6$ km; $L^* = 6{\cdot}32$ km is the reversal distance at which the t change of field is zero. For $L \leqslant L_1$ and $L \geqslant L_2$ a simple field change results.

Thus, depending on the location of the recording stations relative to the discharge, e two records may both show a simple field change or a Type I field change, or one may cord a simple field change and the other a Type I field change, as has been found. The cases noted previously of the reversal effect for simple field changes are special cases which one station was located closer than the inner critical distance for each discharge d the other station located beyond the outer critical distance. For a particular separation



gure 3. Variation of field change with distance for: W, positive charge lowered; X, negative charge raised; Y, negative charge lowered; and Z, positive charge raised.

D, of the recording stations, it is possible to record the reversal effect with simple-type field changes for intracloud discharges for which the vertical movement does not exceed $D/2\frac{1}{2}$. Since $D = 13{\cdot}2$ km in the present case, the condition reduces to $(H - h) \leqslant 9{\cdot}3$ km.

The sense of the turning point indicates the sign of charge moved, whatever the direction of movement; a maximum results from movement of positive charge and a minimum from movement of negative charge. There are four possible combinations of sign of charge and vertical direction of motion. The example of Fig. 2, for the case of a positive dipole in which the negative charge is raised, we shall call type X. We may also have the case in which the positive charge is lowered; this we call type W. There are the corresponding types for negative dipoles : Y, negative charge lowered and Z, positive charge raised. As shown in Fig. 3, the variation of field change with distance from the discharge is different for the four types. The classification of field changes of Type (A to E) according to sign of the net change and sense of the turning point is based on Wormell (1939). The simple field changes (P and N) are labelled according to the sign of the net change. Thus, when observations are available from two stations simultaneously and, in addition, it is known which station is closer to the discharge, we may determine the vertical direction of motion of the charge.

In the present investigation 54 discharges which gave a Type I field change at one or both stations were recorded. For 29 of these the closer station can be identified from the records of the local weather stations. When the direct observations do not enable the closer station to be determined unambiguously, another method is available. It is seen in Fig. 4 that with increasing distance from the discharge the amplitude of the field change (the maximum change of field) decreases almost continuously. There is some possibility of ambiguity when one recording station is close to the reversal distance (L^*) but, with care in applying the rule, we may assume that the field change with the greater amplitude is the closer to the discharge. This rule has been found to be true

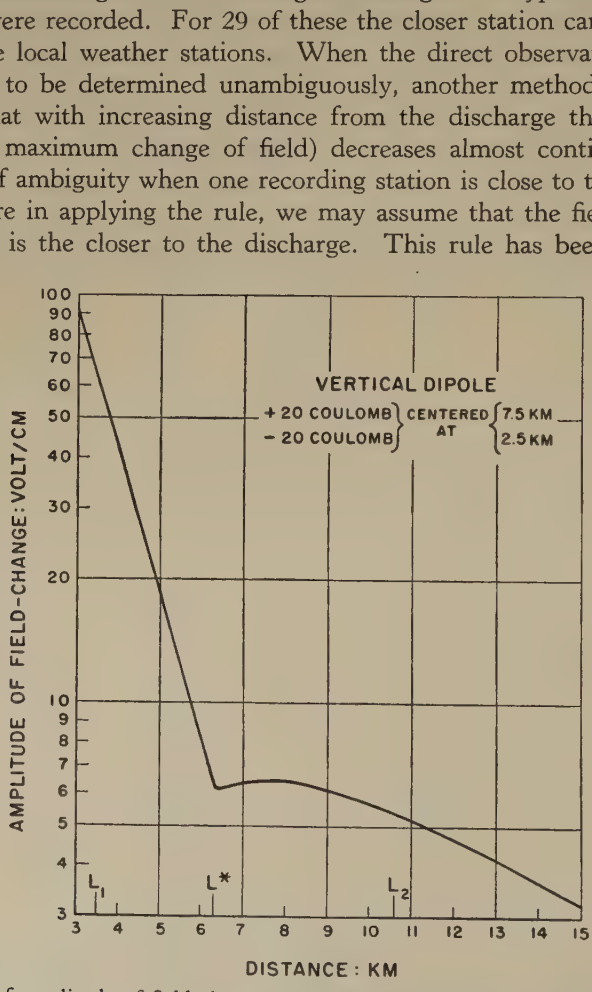


Figure 4. Variation of amplitude of field change with distance, computed for the same charge movement as in Fig. 2.

r the 29 cases in which the closer station could be determined by direct observation. It s, therefore, been used to establish the type for each of the 54 discharges. These were corded on seven days. The resulting frequency for the four types is given in Table 1. he figures indicate that the cloud discharge is of a positive dipole more frequently than a negative dipole by a factor of 2·6 and that for either polarity of the dipole the discharge volves the raising of charge more frequently than the lowering of charge, by a factor 3·3 for positive dipoles and 1·5 for negative dipoles.

TABLE 1. CHARGE MOVEMENT IN CLOUD DISCHARGES

Polarity	Type	Movement	Number
Positive dipole	W	Positive lowered	9
	X	Negative raised	30
Negative dipole	Y	Negative lowered	6
	Z	Positive raised	9
			54

The actual distances of the discharges from the recording stations are not known. owever, in determining the direction of movement of charge, using Fig. 3, the individual eld changes were located relative to the inner and outer critical distance (L_1 and L_2) nd the reversal distance (L^*) appropriate to each discharge. Thus, the field changes ay be grouped in four ranges of distances: (a) $L \leqslant L_1$; (b) $L_1 < L < L^*$;) $L^* \leqslant L < L_2$; and (d) $L_2 \leqslant L$. The distribution of the amplitude of field change for ch of these ranges of distance is given in Table 2 for the 54 discharges which gave a ype I field change at one or both stations, together with the 11 discharges which gave mple field changes with reversed signs at the two stations. Equal intervals of amplitude n a logarithmic scale are found to give roughly symmetrical distributions. Within each nge of distance there is a wide spread of values.

TABLE 2. DISTRIBUTION OF AMPLITUDE OF FIELD CHANGE

Distance	Field change (v/cm)							Total
	0·1–0·32	0·32–1·0	1·0–3·2	3·2–10	10–32	32–100	100–320	
$L \leqslant L_1$	0	0	0	8	13	13	1	35
$L_1 < L < L^*$	0	1	8	15	4	1	0	29
$L^* \leqslant L < L_2$	1	5	9	10	4	4	1	34
$L_2 \leqslant L$	2	5	14	4	6	1	0	32

It may be seen in Fig. 4 that the amplitude of those field changes recorded between e reversal distance and the outer critical distance, group (c), is insensitive to change in stance for any given dipole. Thus, the large range of values of amplitude in this group nnot be due to variation in distance and probably represents variations in the dipoles, magnitude, height and inclination. The amplitude observed at the reversal distance ill increase directly as the magnitude of the charges, and will be greater for dipoles lower the cloud. For inclined dipoles the amplitude will be greater or less depending on the rection of the recording station. As an example, we take the dipole model used earlier nd displace one charge horizontally by 5 km, giving a dipole with an inclination of 5 degrees. The reversal distance now depends on direction; at the closest point the mplitude of the field change is 53 volt/cm and at the most distant point it is 0·40 volt/cm. or the original vertical dipole the value was found to be 6·2 volt/cm. It thus appears at the three factors are sufficient to account for the wide range of values observed.

The amplitude of field changes in the range $L^* \leqslant L < L_2$ are plotted in Fig. 5 (a) together with the separate observations for (b) positive dipoles and (c) negative dipoles. The distribution for positive dipoles is centred at about 3 volt/cm and that for negative dipoles at about 20 volt/cm. Assuming from Pierce (1955) that the average positive and negative dipole moments destroyed in cloud discharges are equal within a factor of two we deduce that within the cloud, on the average, the discharge of positive dipoles takes place higher than does that of negative dipoles. Wormell (1939), examining the prevailing sign of field changes, has arrived at the same conclusion.

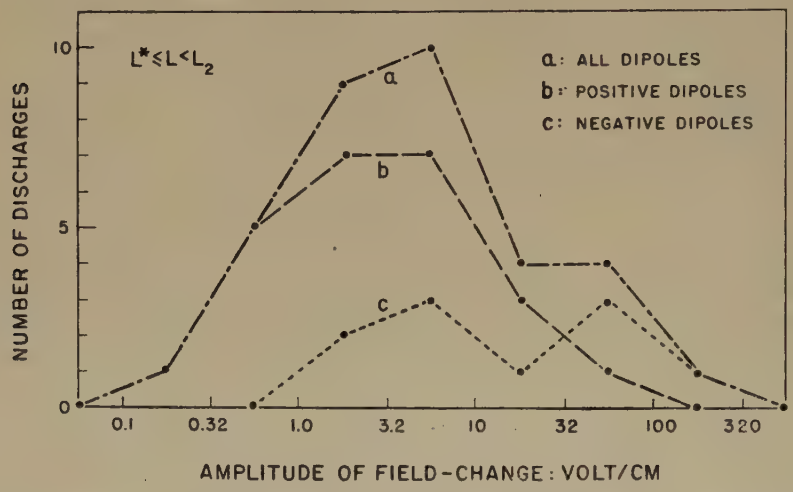


Figure 5. Frequency distribution of amplitude of field change for discharges between the reversal distance (L^*) and the outer critical distance (L_2).

We have shown earlier in this section that Type I field changes should be observable only over a limited range of distance from the cloud discharge. For the particular model used to illustrate the analysis, this range is from about 3 km to 10 km. Since in the present experimental arrangement discharges are recorded at distances up to about 25 km, it would be expected that the proportion of slow field changes which are complex of Type I would be about 15 per cent. Thus, the observed proportion, 14 per cent, is reasonable. Owing to variations between individual dipoles, indicated earlier, the proportion is not zero beyond 10 km from the discharge, but can be expected to decrease with increasing distance. This explains why Pierce (1955), recording more distant discharges, does not find complex slow field changes of Type I in significant numbers.

6. Complex slow field changes of Type II

There is considerable variation in the relative magnitude of the three components of this type, as can be seen in the examples of Fig. 1. In some cases the amplitude exceeds the net change of field.

In 86 per cent of the Type II field changes the net change of field is negative, ranging from $-$ 0·13 to $-$ 14·5 volt/cm. The values are distributed symmetrically, on a logarithmic scale, about a mean of 2 volt/cm. The 14 per cent having positive net change of field range from 14 to 48 volt/cm, with a mean of 30 volt/cm. It is interesting to note that these mean values for Type II complex slow field changes are approximately the same as the mean values of the positive and negative simple slow field changes which showed the reversal effect.

Comparing the simultaneous records at the two stations, it is found that when a ype II field change is observed at one station, the other station may record a field change f the same type or a simple field change and, for either case, the sign of the net change ay be the same or opposite. Also, when Type II field changes are recorded at both ations, the relative magnitudes of the three components are generally different.

When Type II field changes are recorded by both stations, but with opposite signs f net change, the corresponding turning points are in the opposite sense, contrary to ie experience with Type I field changes. This indicates that each turning point is not ue to a single vertical movement of charge. This is supported by the observation by ierce (1955) that field changes of Type II occur in significant numbers at distances ell beyond the reversal distance.

It is appropriate to consider at this point a feature noted by Pierce (1955) that the imple slow field changes may consist of sections of slow change separated by intervals f even slower change in field. There are five cases in the present series of observations hen a simple change with two sections was recorded at both stations, and one case of simple change with three sections at both stations. In six cases, however, a simple hange with two sections was recorded at one station, and a Type II field change at the ther. In three other cases a Type II field change was recorded simultaneously with a imple field change of one section; Fig. 1 (f) is an example. In these cases it is seen, on areful examination, that the simple field change has slight changes in slope corresponding ith the turning points on the Type II field change recorded at the other station.

We may expect that the interpretation of Type II field changes is to be found in a ore complicated charge movement than was assumed for Type I field changes, perhaps multiple discharge within the cloud. In this case, Type II field changes would be observed oth close to and far from the discharge, and subject to the reversal effect. At some ntermediate range of distance, the two turning points of the multiple discharge will be ompounded with the single turning point of a Type I field change to give one of the ore complex field changes which have been found occasionally.

It is planned to investigate directly the existence of multiple cloud discharges by ecording the illumination simultaneously with the field change, and we shall not consider hem further in this paper.

7. Conclusions

This investigation has shown that the simultaneous recording of slow electric-field hanges at two suitably separated ground stations is a powerful method of studying ntracloud lightning discharges. It has been found possible to demonstrate the reversal ffect for individual discharges and deduce something of the movement of charge during he discharge. We have shown that the majority of slow field changes can be explained atisfactorily by the uniform movement of a single charge within the cloud. The remainder, bout 17 per cent, implies a more complicated charge movement, perhaps involving nultiple discharges.

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Barometric tendency and weather at Changi, Singapore

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Summary

Barometric tendencies at Changi are ' corrected ' for the diurnal effect and then examined in relation to the occurrence of rain in the following period. It is shown that the three-hourly corrected tendencies are of no value in forecasting rain.

1. Introduction

Scott (1955) has investigated weather trends at Singapore in relation to a 24-hr tendency parameter. As most of the rainfall of Singapore is of short duration (less than three hours) it is reasonable to suppose that the rain-making processes are of similar short duration. It was therefore thought worth while to investigate the relationship between the weather in three-hourly periods and the relevant three-hourly tendencies. This paper describes this investigation.

It is generally accepted that in equatorial regions the surface isobars are not representative of low-level wind flow. Since the actual wind flow is the factor dominating the weather it follows that weather and isobars are not closely related and forecasters can make little use of surface isobaric charts in equatorial forecasting. If we are presented with a series of isobaric charts each of which is not related to the weather it is reasonable to suppose that the difference between these charts (i.e. the tendency) will also bear little relation to the weather, and this applies whatever the interval between the charts. Watts (1955) in his *Equatorial weather* refers to the lack of value of pressure tendencies as a forecasting tool. 24-hr tendencies are usually referred to, but it is suggested here that at Singapore a 24-hr period is much longer than the usual rain-making process and that it is therefore quite likely to miss the evidence for such a process.

2. Data used

The data used were for the two years 1954 and 1955, taken from the records of observations at Changi at the extreme eastern tip of Singapore Island. To adjust the tendency for diurnal effects, pressure data obtained from the Malayan Meteorological Service relating to the ten years 1931-1940 inclusive were used. From July 1937 these data related to Kallang Airport and prior to that date to an observing station about one mile distant.

3. Diurnal tendency

The diurnal tendencies shown in Table 1 were obtained by differencing the 10-yr mean pressures referred to above.

TABLE 1. Diurnal tendencies (in mb) – Singapore (based upon mean pressures for 1931-1940 inclusive, supplied by the Malayan Meteorological Service)

GMT	J	F	M	A	M	J	J	A	S	O	N	D
0000	1·0	0·9	1·0	1·1	1·0	0·9	0·7	0·8	1·0	1·1	1·1	1·1
0300	0·9	1·0	1·1	0·9	0·8	0·7	0·8	0·9	0·9	0·8	0·8	0·9
0600	− 1·8	− 2·0	− 2·0	− 2·1	− 1·8	− 1·4	− 1·5	− 1·5	− 2·0	− 2·1	− 2·1	− 1·8
0900	− 1·6	− 1·8	− 1·9	− 1·6	− 1·5	− 1·5	− 1·4	− 1·6	− 1·5	− 1·5	− 1·4	− 1·5
1200	1·4	1·0	1·3	1·1	1·1	0·9	0·9	0·9	1·1	1·4	1·5	1·4
1500	1·6	2·2	1·9	2·0	1·8	1·8	1·6	1·7	1·8	1·7	1·6	1·5
1800	− 0·6	− 0·3	− 0·3	− 0·5	− 0·5	− 0·4	− 0·3	− 0·4	− 0·6	− 0·8	− 0·6	− 0·4
2100	− 0·9	− 1·0	− 1·1	− 0·9	− 0·9	− 1·0	− 0·8	− 0·8	− 0·7	− 0·6	− 0·9	− 1·2

Individual tendencies might be expected to differ from these mean values according to the eather. However, Haurwitz (1955) has investigated the thermal influence on the daily pressure ave at Bermuda. His introductory abstract states ' The effect of clear and cloudy days, and f days with large and small temperature ranges, on the daily pressure variation at Bermuda is ıvestigated. It is shown that the various harmonic components are not significantly affected by ıe different weather conditions, contrary to what has been found for continental weather ations. Such an independence from the local weather situation is to be expected over the ceans.' Changi is very much under oceanic influences and it will be assumed that the findings r Bermuda can be reasonably applied to Changi.

4. Analysis

December 1954 was a very exceptional month. 36 in. of rain fell, this being the highest ver recorded at Singapore and approximately three times the December average. In relating ndency to weather it was therefore thought that a suitable beginning could be made with this ıonth.

Each of the three-hourly tendencies (00 to 03 GMT, 03 to 06 GMT etc.) was corrected for iurnal effect using Table 1; e.g. a tendency of 20 at 0000 GMT was treated as + 09. The ainfall in each three-hour period was written down, a trace being treated as nil. Each period ıen falls into one of the following categories :

(a) No rain – next period with no rain.
(b) Raining – next period the same or less rain.
(c) No rain – next period raining.
(d) Raining – next period with more rain.

(c) and (d) can be treated as periods in which a rain-making process was in operation while a) and (b) are periods when probably no such process was in action.

Table 2 is a frequency distribution for all the December 1954 tendencies (corrected).

TABLE 2. All December 1954 tendencies

Central corrected tendency (tenths of mb)	− 20	− 15	− 10	− 5	0	5	10	15	20	Total
Observed frequency	1	3	18	52	93	56	17	8	0	248
Normal frequency	0	4	21	58	82	58	21	4	0	248

The mean and standard deviation of the observed frequency are + 0·26 and 5·87 tenths espectively. The normal frequency distribution for mean zero, and standard deviation 5·87 iven in Table 2 is clearly a satisfactory fit.

Table 3 gives the distribution in groups (a), (b), (c) and (d).

TABLE 3. December 1954 tendencies

Central corrected tendency (tenths of mb)		− 20	− 15	− 10	− 5	0	5	10	15	20	Total
Frequency	(a)	1	2	11	25	58	35	8	3	0	143
	(b)	0	0	5	10	19	12	6	4	0	56
	(c)	0	0	2	9	10	3	1	0	0	25
	(d)	0	1	0	8	6	6	2	1	0	24

The χ^2 test shows that any of the frequency distributions (a), (b) or (c) + (d) could well ave been a random sample from the normal frequency curve representing all tendencies, and he groups are therefore not distinguishable. Given any observed (corrected) tendency we are hen quite unable to say what the weather in the following tendency interval will be.

This conclusion is based upon one very exceptional month. Although the rain-making process in this month was very active it might be argued that it would have been preferable to have studied more normal conditions. Accordingly an investigation of the whole of 1954 and 1955 was undertaken, attention being limited to three-hour periods in which 5 mm or more of rain were recorded, providing less than this amount had fallen in the preceding period. That tendency in the period preceding such rain can be thought of as involved in a rain-making process – it will be called T_1. The tendency during the period can be thought of as involved in a rain-making process providing the rain in the following period increased. This tendency will be called T_2. The tendency during the period can be thought of initially as involved in a 'fine-making' process providing the rain in the following period decreased. Such tendencies will be called T_3.

Table 4 shows the frequency distributions of T_1, T_2 and T_3 for the two years 1954 and 1955.

TABLE 4.

Central corrected tendency	−25	−20	−15	−10	−5	0	5	10	15	20	25	N
T_1				7	37	51	50	16	11	3	–	175
T_2					4	8	12	9	8	2	–	43
T_3	1	1	3	9	18	28	39	20	12	0	1	132
$T_1 + T_2$				7	41	59	62	25	19	5	0	218

For $T_1 + T_2$ Mean = + 3 tenths mb. Standard deviation = 6·6 tenths mb.
For T_3 Mean = + 2·5 tenths mb. Standard deviation = 7·9 tenths mb.

In both cases the mean is significantly positive. The standard error of the difference between these two means is $(\sigma_1^2/N_1 + \sigma_2^2/N_2)^{\frac{1}{2}} = 0{\cdot}82$, and the difference between them is 0·5; the two means are not significantly different. The tendencies before rain, or during rain but preceding heavier rain, cannot therefore be distinguished from the tendencies during rain which was followed by its decreasing or cessation. In this limited sense the rain-making and 'fine-making' process are not distinguishable and this confirms the conclusions found for December 1954. Although the tendencies for periods of no rain followed by no rain have not been examined it can be inferred that the mean of such tendencies will not differ significantly from zero. It then follows that over the 2-yr period there was a significant difference between the tendencies for fine periods followed by fine periods and fine periods followed by rain. The latter have in the mean, small, but significantly positive, tendencies.

5. Conclusions

(a) Over two years the mean of tendencies in fine periods followed by fine periods does not differ significantly from zero.

Over the same two years, tendencies preceding rain or during rain which became heavier are not distinguishable from tendencies during rain which became less. Both groups are in the mean significantly positive (+ 2·5 to 3 tenths mb) and in this limited sense a 'fine-making' and rain-making process are not distinguishable. The tendencies T_2 and T_3 however are tendencies during rain and their significantly positive values may be associated with the atmospheric cooling associated with the rainfall. The tendencies T_1, that is in fine periods preceding rain, are in the mean significantly positive. When reduced to a single month all significant differences may vanish. When reduced to a single tendency nothing whatsoever can be inferred. From the immediate forecasting point of view therefore three-hourly corrected tendencies are of no value whatever.

(b) Although three-hourly corrected tendencies have no immediate forecasting value the results found are significant to the general problem of equatorial forecasting in this region. We have found that any weather can be associated with any tendency but that rainy periods are on balance associated rather more with rising pressures than with falling pressures. It would be tempting to fit this into the patterns suggested by Riehl (1954) in considering waves in the easterlies. However, the dominant flow at Singapore at 10,000 ft during the whole year is westerly and conditions suitable for the formation of such waves in this region are only occasionally experienced.

All the corrected tendencies are quite small. For any period this may be either because ere is no particular atmospheric process operating or because substantial convergence is being lanced by substantial divergence. In rain-making periods here, substantial low-level conver- nce is being balanced by substantial upper-level divergence. It is suggested that which of ese initiates the process is revealed by the sign of the balance. When the balance is positive w-level convergence initiates the process and when the balance is negative, high-level divergence itiates. Considering only those fine periods which were followed by rain (T_1 in Table 4), had a negative balance and 80 a positive balance. This suggests that at Singapore by far the ost important initiator of rainfall is low-level convergence. This convergence is revealed by e low-level wind-flow charts which consequently are of major importance in forecasting here.

6. Acknowledgments

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Anisotropic high-level turbulence

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Summary

A study is made of several cases of high-level turbulence in which the intensity varied with the direction of flight of the aircraft. The direction of maximum turbulence is found to coincide fairly well with the direction of the contour lines. A theory is developed which attributes this type of turbulence to quasi-horizontal gusts associated with strong horizontal wind shear. Gravity waves are examined as a possible alternative explanation and tentatively rejected.

1. Introduction

Bannon (1951) noted that the high-level turbulence affecting the motion of aircraft was frequently associated with strong horizontal wind shear. It would seem logical that turbulent energy due to horizontal shear should be organized largely in the form of quasi-horizontal gusts. Nonetheless, many authors have doubted that such gusts can be strong enough to cause aircraft bumpiness.

This paper cites cases in which aircraft bumpiness varied markedly in intensity with direction of flight. It will be argued that the only adequate explanation of this phenomenon necessitates the assumption of anisotropic, quasi-horizontal turbulence. Evidence will be presented that horizontal gustiness may readily be anisotropic at high levels, and, when this is so, that the largest component of the mean gust velocity will usually act in the direction of the mean wind. Horizontal gusts can cause vertical accelerations on an aircraft by varying the instantaneous airspeed and thus increasing or decreasing the lift. Owing to this mode of action of horizontal gustiness on an aircraft, the gust velocities in the direction of flight will be effective in producing vertical accelerations, whereas the cross-wind gust velocities will be relatively ineffective. Thus, turbulence which is strongly anisotropic in the horizontal will result in marked variation with direction of flight in the intensity of the aircraft response. The writer can think of gravity waves of short wavelength as the only other mechanism which would explain the observed effect and this approach has several major drawbacks, as will be discussed in Section 3 (b).

2. Aircraft investigation

(*a*) *Description of results*

The observational programme was carried out by pilots of the Royal Canadian Air Force engaged in routine exercises and training flights in jet aircraft. When feasible, the pilots investigated areas of high-level turbulence to see if there was any variation in turbulence intensity with change in flight direction. In interesting cases as many headings as possible were to be flown, and an attempt made to return to the original heading to see if the original intensity reappeared. Six reports of occurrences of this phenomenon were received and these are listed in Table 1 together with pertinent details.

The manoeuvres of the aircraft investigating the above cases probably extended over distances of the order of 15-30 mi and through layers of 1-2,000 ft in depth. Occasionally, patches of high-level turbulence are of this size (Clodman 1953) so that spatial variation should first be considered as a possible cause of the intensity changes. However, a study of the pilots' reports makes fairly clear that spatial variation as the major effect can be ruled out except, perhaps, in cases 2 and 4. These latter cases are ambiguous but in other instances it would appear highly unlikely that the aircraft were crossing the boundaries of the turbulent patches every time they changed direction. Also the relationship between the directions of maximum and minimum turbulence, and between these and the wind field, is difficult to explain on the basis of spatial variation.

TABLE 1. OBSERVATIONS OF ANISOTROPIC TURBULENCE

se	Date and time	Place	Height (ft)	Summary of pilot's remarks
	14 Aug. 1953 1330 GMT	30 mi S and W of North Bay, Ontario	25,000	Light to moderate turbulence on a heading of 220° Mag., a 90° turn toward 130° caused turbulence to fade and disappear by the time 130° was reached. A turn to 220° was made and the turbulence reappeared. Turning to 130° the turbulence faded though not as markedly as before. In a 180° starboard turn the turbulence intensified to 220° and faded to very light on 310°
	27 Oct. 1953 1700 GMT	100 mi W of Winnipeg, Manitoba	35,000 to 36,000	Moderate turbulence on northerly heading becoming light on north-westerly, with no turbulence on heading from north-west through to south. Aircraft was in a climbing turn during manoeuvre
	3 Dec. 1953 1615 GMT	55 mi NW of Macdonald, Manitoba	28,500 to 35,000	Heading which gave minimum intensity of turbulence at 35,000 ft was 270° Mag. and this veered gradually to 300° Mag. on descent to 28,500 ft
	3 Feb. 1954 1515 GMT	Gimli, Manitoba	24,000	Generally moderate turbulence on all headings but maximum on southerly heading
	19 Feb. 1954 2100 GMT	40 mi NE of Gimli, Manitoba	21,000 to 23,000	Severe turbulence on north-easterly heading becoming very light on south-easterly heading and severe again on south-westerly heading
	3 Mar. 1954 1915 GMT	30 mi N of Macdonald	17,000 to 19,000	Turbulence most severe on north and south headings; disappeared with 90° change of direction

(*b*) *Analysis of the reports*

Upper-air charts were carefully constructed by differential-analysis techniques for the times ıd levels appropriate to the cases of Table 1. Where necessary, interpolation in time and/or ace was carried out to ensure a close approximation to the conditions that existed during the rbulence incidents. The wind and pressure fields associated with each of the cases were then ıdied by means of these charts. The results of this analysis are given in Table 2. Case 3 volved an investigation through a deep layer and this was handled as two occurrences, one at e bottom and one at the top of the layer.

ABLE 2. ANALYSIS OF WIND FIELD IN CASES OF ANISOTROPIC TURBULENCE (values in columns 2, 3 and 4 are given in degrees true)

1	2	3	4	5	6	7	8
ıse	Minimum turbulence	Maximum turbulence	Contour directions	Difference between columns 3 and 4	Horizontal wind shear	Vertical wind shear	Remarks
	120 and 300	210	250–070	40	Strong anticyclonic	Weak	Polar frontal surface below at about 750 mb. Jet stream about 100 mi NNW
		360	350–170	10	Probably strong cyclonic (vicinity of weak jet)	Weak	Polar front just west at 300 mb. Weak jet stream in vicinity of incident so shear may not be cyclonic
a).	310	(220)	225–045	5	Strong cyclonic	Very weak	Polar front nearby. Just below tropopause. Jet stream 50 mi SE
b).	280	(190)	205–025	15	Strong cyclonic	Weak (negative)	Just above tropopause. Jet axis about 50 mi SE
	–	180	310–130	50	Probably strong cyclonic (vicinity of jet)	Fairly weak	Jet axis and polar front in vicinity of incident
	135	225 and 045	225–045	0	Strong anticyclonic	Fairly weak	Jet axis just to NW
	090 and 270	360 and 180	010–190	10	Strong anticyclonic	Very weak	Thermal trough

Although there were no reports of the directions of maximum turbulence in cases 3 (a) and 3 (b), calculated figures are entered in column 3 for comparison purposes. These figures represent the normal to the direction of minimum turbulence since the directions of maximum and minimum intensities were found to be perpendicular to each other when both were reported. Column 5 gives the absolute difference in degrees between the direction of maximum turbulence and the direction of the contour lines. It is apparent that there is a close relationship between these two parameters, particularly when one considers the accuracy attainable in these measurements.

Case 4 involves a turbulent area in which the pilot encountered moderate turbulence on all headings with a maximum in a southerly direction. Apparently there was little variation in the intensity of turbulence with no second maximum on a northerly heading nor minima when flying east or west. This suggests that spatial rather than directional variability was involved in this case.

In case 2 the pilot reported maximum turbulence on a northerly heading but did not find the turbulence to re-intensify when he turned toward the south. However, he was climbing during the manoeuvre and may have climbed out of the turbulent layer before entering the southerly heading.

In drawing upper-level charts, a certain amount of smoothing is inevitable so that it is impossible to secure an accurate measure of the horizontal wind shear. Nevertheless in all the cases studied, strong horizontal shears are indicated. This fits in with our concept of the causative mechanism, although it is conceivable that strong quasi-horizontal gusts may result from other turbulence-producing mechanisms. The vertical wind shears shown in column 7 were secured from radiosonde and rawinsonde data and these shears are limited by the resolution of the measuring system. However the shear intensities were all weak when compared to average values secured by these techniques. It would therefore appear unlikely that vertical shear plays a part in producing the type of turbulence being investigated.

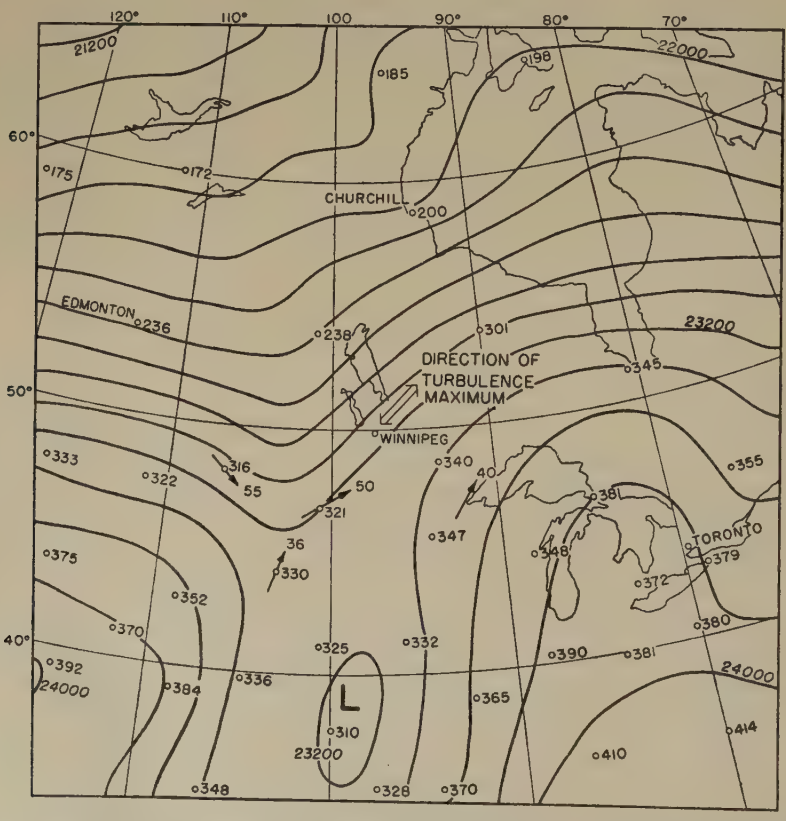

Figure 1. For case 5. 400-mb chart 19 February 1954, 2100 GMT. Height data interpolated between 1500 GMT and 0300 GMT reports. Wind speeds in knots given beside wind direction arrows.

Fig. 1. shows the chart associated with case 5. This incident took place at 22,000 ft at 00 GMT, 19 February 1954. To approximate the conditions encountered, the 400-mb chart as built up from the 2100 GMT surface chart. The 2100 GMT upper winds were used and the ight data were interpolated between the 1500 and 0300 GMT radiosonde reports.

3. Cause of directional effect

(*a*) *Horizontal gusts*

In Section 1 we suggested that quasi-horizontal, anisotropic turbulence can cause a direction-y variable response in aircraft. The observational data of Section 2 fit in with this picture nce strong horizontal shear was involved in each of the cases. Moreover, the uniform absence strong vertical shear is significant. Vertical shear would be likely to produce vertical gusts nich are non-directional in their effect on aircraft and these would therefore tend to obscure e directional variability due to the horizontal gusts.

In order to complete the picture we will demonstrate that horizontal gustiness associated th strong horizontal shear may be anisotropic, and, when this is so, that the maximum gustiness ll usually be in the streamline direction and the minimum gustiness at right angles to it.

Let us assume that a parcel is accelerated by a perturbation due to turbulent motion, and us is displaced in a quasi-horizontal direction from its normal streamline motion. The gust locity of this disturbed parcel will be made up of two components, one due to the initial rturbation and the other due to its being displaced to an area of different mean wind strength. hus the gust velocity in the direction of the mean wind will depend on the initial perturbation d the shears $\frac{\partial u}{\partial y}$ and $\frac{\partial u}{\partial x}$, where u and x are in the direction of the mean wind. The cross-nd gust velocity will be a function of the initial perturbation and the shears $\frac{\partial v}{\partial y}$ and $\frac{\partial v}{\partial x}$. Since $\frac{\partial u}{\partial y}$ is often much greater than the other shears it would appear likely that, not infrequently, the ust velocity in the direction of the mean wind will be significantly greater than the cross-wind mponent. The reverse situation with $\left|\frac{\partial v}{\partial x}\right| \gg \left|\frac{\partial u}{\partial y}\right|$ could occur near the centre of an in-nse low with a small value of horizontal shear, but this should be a relatively rare phenomenon.

The above elementary argument does not take into account the accelerational forces acting the parcels. However, by assuming large but realistic values of wind shears, it was possible use Godson's (1950) study of the motion of such disturbed parcels to indicate the validity the above conclusions. The eddies dealt with in this extension of Godson's theory were much rger than those to which aircraft respond, since little is known of the magnitude of the relevant ears involved in the smaller scales of turbulence. Nevertheless, the theory also indicates that rge eddies which are anisotropic will tend to produce smaller eddies which are similarly aniso-opic. This follows since the large-scale gust velocities will act to produce small-scale shears d if the gusts vary in intensity with direction so will the resultant shears. It is not to be expected at this anisotropy will extend to the smallest turbulent motions but it does suggest that in e degradation of high-level quasi-horizontal turbulent motion, the tendency toward isotropy ay not be as rapid as at low levels. Even at low levels, for eddies of the order of 50 m, isotropic rbulence is not the universal rule and anisotropy has been detected (Durst 1955).

The argument of this section can only indicate the probability or possibility of the existence anisotropic turbulence at high levels. It should be borne in mind that the major evidence, so r as this paper is concerned, for the existence of quasi-horizontal, anisotropic turbulence is herent in the observations of Section 2.

(*b*) *Gravity waves*

Only gravity waves suggest themselves as another possible mechanism which could directly use a directionally-variable response in aircraft. Such waves may occur as travelling waves at surface of discontinuity or they may be standing waves associated with orographic features. he particle motion associated with such waves will have both vertical and horizontal com-onents. Since the resultant gust velocities in the vertical and horizontal would be of the same

order, the major effect on aircraft would be due to the vertical component. Thus, if the wave lengths are small enough, an aircraft flying across the waves (from crest to crest) would experience a rapid bumpiness. While flying roughly parallel to the waves there will be only a slow change between motion up and down. Gravity waves of the correct size would therefore result in turbulence whose intensity would change with direction of flight. Moreover, the direction of maximum reported intensity would be at right angles to the waves and this would be in the direction of the mean wind. We cannot, therefore, *a priori*, rule out such waves as a possible explanation for the directional effect.

It is generally conceded that large gravity waves can cause aircraft bumpiness by creating intense local shear zones. However, to cause the phenomenon described in Section 2, waves would have to have a wavelength of only a few hundred feet and no evidence for the existence of extensive and persistent trains of such small wavelength has as yet been presented.

If standing waves were involved it would seem necessary that there should be associated a substantial topographic feature. Under such circumstances, the waves would have too large a length to give the desired effect although it is conceivable that such waves may degenerate to waves of smaller wavelength before the energy is entirely dissipated. A careful examination of the topography associated with each of the cases was carried out and this revealed only minor variations in terrain, none of which were properly aligned with the wind field.

The problem of travelling waves on a surface of discontinuity of the first order has not been solved, and there is some doubt that such waves can exist with any persistence. Nevertheless, a study of the cases involved was carried out and this indicated that only four of the seven flights were associated with discontinuities in the way of a tropopause or a front.

If gravity waves were involved one would expect a change in bump frequency with change in direction of flight. A tendency toward rolling and tossing would also be expected. Neither of these effects were mentioned by any of the pilots whose reports were studied above. On the basis of the above discussion it would appear unlikely that small gravity waves were the direct cause of the bumpy flight in the cases studied.

4. Conclusions

Aircraft bumpiness, on occasion, may vary in intensity with the aircraft heading and the maximum bumpiness occurs approximately in the streamline direction. Little can be said on the frequency of occurrence of this phenomenon although it appears to be associated with strong horizontal shear. Since quasi-horizontal, anisotropic turbulence is apparently the causative mechanism, we are forced to conclude that horizontal shear can result in quasi-horizontal gustiness and further that horizontal gusts can be strong enough to give fairly intense aircraft bumpiness. Bannon (1951) found that a significant amount of high-level turbulence was associated with strong horizontal shear. Perhaps only a small percentage of these cases was associated with marked directional variation. However, it appears likely that quasi-horizontal gusts play a much more important role in high-level turbulence than has hitherto been thought.

Two corollaries of the above conclusions deserve mention. Firstly, it is evident that change of direction may, on occasion, be a useful technique for reducing aircraft bumpiness at high levels. Secondly, when horizontal gusts are involved the practice of reducing airspeed on encountering turbulent air will not be effective. The number of gusts per minute will be reduced by this manoeuvre but the percentage effect on air speed of the individual gusts will be increased.

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The intensity of turbulence in convective clouds

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(Manuscript received 23 July 1956)

Summary

Calculations of collision rates between cloud droplets in turbulent air show that this would be an ective precipitation mechanism in cumulus if the r.m.s. turbulent acceleration were about 3 times that e to gravity. An estimated value, based on published records of flights through thunderstorms, is about e-sixth of gravity. In the absence of more direct evidence, it seems that turbulence is not an important tor in convective precipitation.

1. Introduction

East and Marshall (1954) suggested that turbulence might have an important role in the owth of cloud droplets, and they calculated collision rates between droplets in turbulent air; e importance of the effect as a precipitation mechanism could not be estimated at that time r lack of suitable observations. In this paper the modification of drop-size distributions by the tion of turbulence is computed, and the intensity of turbulence is estimated from indirect servations.

2. Frequency spectrum of turbulence

The behaviour of a cloud droplet of radius r in turbulent air is most conveniently expressed its ' time constant '

$$\tau_r = 2\rho_s r^2/9\eta_a$$

here ρ_s = density of droplet and η_a = dynamic viscosity of air which is taken as $1{\cdot}72 \times 10^{-4}$ cm^{-1} sec^{-1}. It was shown (East and Marshall 1954) that turbulent acceleration A of the air oduces a relative velocity U between a drop, radius s, and a droplet, radius r, where

$$U(t) \simeq (\tau_s - \tau_r)\, A(t) \quad . \quad . \quad . \quad . \quad (1)$$

ovided the frequency spectrum of the turbulent velocity satisfied the condition that above a equency $\omega_H/2\pi$ the energy should fall off rapidly, and that at this frequency

$$\omega_H \tau_s \ll 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

he Reynolds number R in cumulus clouds is so high that the inertial sub-range can be assumed exist (the terminology and notation are those of Batchelor (1953), see also § 5a below). Then e wave-number spectrum falls off as $\kappa^{-5/3}$ up to a wavenumber $1/\eta$, above which it falls off as $^{-7}$ or some steeper law. Here $\eta = \nu^{\frac{3}{4}} \epsilon^{-\frac{1}{4}}$. The frequency spectrum is closely related and also lls off rapidly above an angular frequency v/η, where $v = \nu^{\frac{1}{4}} \epsilon^{-\frac{1}{4}}$. Identifying this angular equency as ω_H, and putting $\nu = 0{\cdot}180$ cm^2 sec^{-1} and $\epsilon = 10^4$ cm^2 sec^{-3} (corresponding to g. 3) we find $\omega_H = 236$ sec^{-1}. Then $\omega_H \tau_s = 1$ for $s = 45\mu$, so that Eq. (1) is valid for esent purposes.

3. Collision rates

For a given drop, radius s, and droplet, radius r, in a steady gravitational field g_E, the relative locity U, Langmuir's parameter $K = U\tau_r/s$ and collision efficiency E have steady values which epend on s, r; call them U_1, K_1 and E_1. When the excitation is random U, K and E are also ndom functions of time. We define $k = U/U_1$, so that $K = kK_1$; k will be called the ' reduced angmuir parameter." Whenever the relative velocity happens to have the value it would have ith gravity alone, $k = 1$.

A drop, radius s, mass m_s, grows by collisions with droplets whose radii lie between r and $r + dr$ at the rate

$$\left(\frac{dm_s}{dt}\right)_r dr = \pi (s + r)^2 \overline{UE} \cdot w(r)\, dr = \pi (s + r)^2 \int_0^\infty UE(K)\, P(U)\, dU \,.\, w(r)\, dr \quad . \quad (4)$$

where $w(r)\, dr$ is the liquid-water content of the droplets (mass per unit volume) and $P(U)\, dU$ is the probability that the relative speed lies between U and $U + dU$. Then

$$\left(\frac{dm_s}{dt}\right)_r dr = \pi (s + r)^2 U_1 \int_0^\infty kE(kK_1)\, P(k)\, dk \,.\, w(r)\, dr \quad . \quad . \quad (4)$$

We can put

$$E' = \int_0^\infty kE(kK_1)\, P(k)\, dk \quad . \quad . \quad . \quad . \quad (5)$$

so that $U_1 E'$ can replace $\overline{UE}$ in Eq. (3). Call E' the mean efficiency for this particular probability distribution.

The steady relative velocity due to gravity g_E is $U_1 = (\tau_s - \tau_r)\, g_E$ if Stokes's Law applies. For a first attempt the horizontal components of acceleration were neglected, and the vertical component $A(t)$ was combined with the effect of gravity. From Section 2 the resulting random vertical relative velocity

$$U(t) \simeq (\tau_s - \tau_r)\, A(t)$$

provided the spectrum of air velocity satisfies certain conditions. Then since $A(t)$ has a Gaussian distribution so has $U(t)$; we take the r.m.s. value of $A(t)$ equal to g_E, so that the r.m.s. value of $U(t)$ is U_1. Then

$$P(U)\, dU = \frac{1}{U_1 (2\pi)^{\frac{1}{2}}} \left[\exp\left\{-\frac{(U - U_1)^2}{2U_1^2}\right\} + \exp\left\{-\frac{(U + U_1)^2}{2U_1^2}\right\}\right] dU,\ U \geqslant 0 \quad . \quad (6)$$

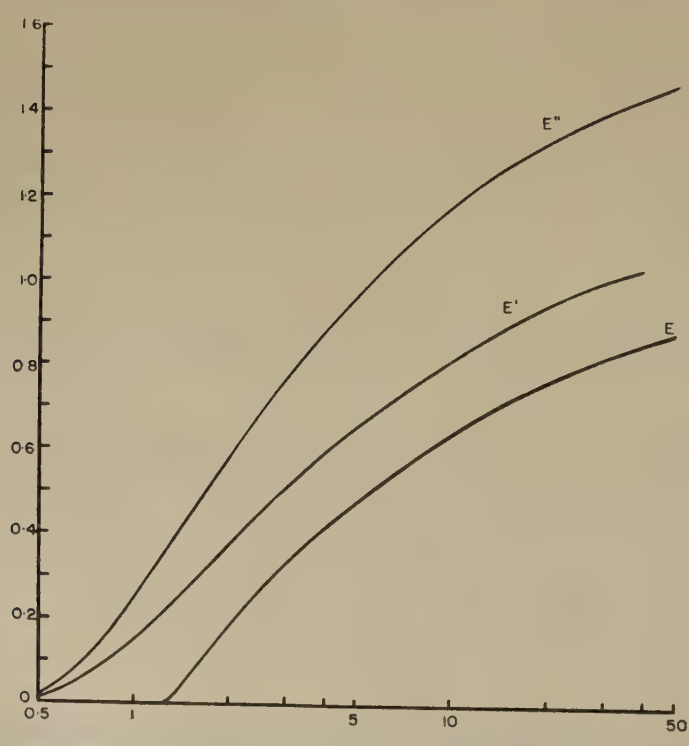


Figure 1. Collision efficiency and mean efficiencies. Curve E : collision efficiency versus Langmuir's parameter $K = U\tau_r/s$. As $K \to \infty$, $E \to 1$. Curve E' : one-dimensional turbulence, $(\overline{A^2})^{\frac{1}{2}} = g_E$: mean efficiency versus K_1, the value which K has when acceleration $= g_E$ (Eq. 5). As $K_1 \to \infty$, $E' \to 1{\cdot}17$. Curve E'' : three-dimensional turbulence, $(\overline{A^2})^{\frac{1}{2}} = 2g_E 3^{\frac{1}{2}}$: mean efficiency versus K_2, the value which K has when acceleration $= 2g_E$. (Eq. 8). As $K_2 \to \infty$, $E'' \to 1{\cdot}60$.

ere U now refers to relative speed rather than velocity. In reduced form

$$P(k)\,dk\,(2\pi)^{-\frac{1}{2}}\,[\exp\{-(k-1)^2/2\}+\exp\{-(k+1)^2/2\}]\,dk,\ k \geqslant 0 \quad . \quad . \quad (7)$$

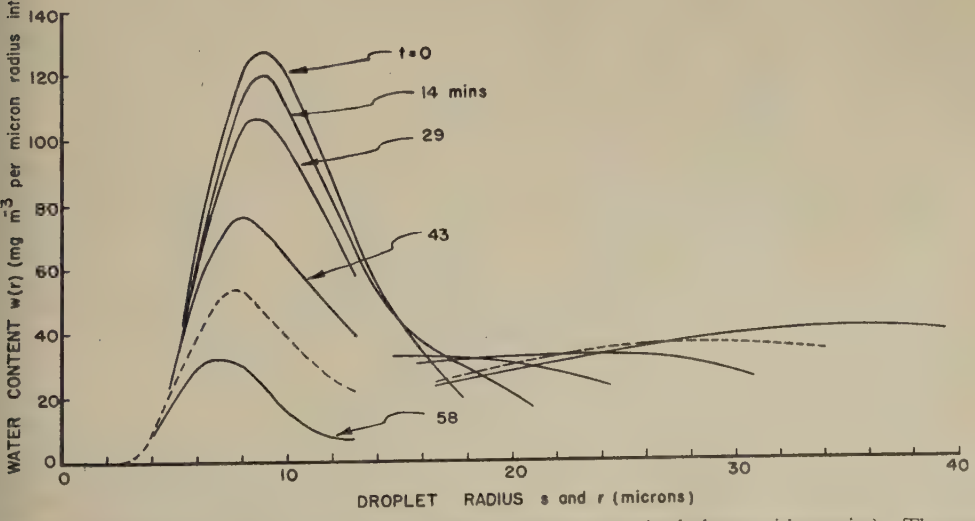
ne above analysis is not accurate if Stokes's Law is not obeyed. However, U_1 is the correct lue for acceleration g_E, and $U(t)$ fluctuates above and below it; by using the reduced form, q. (4), the errors tend to cancel in the integral.

The function E' given by Eq. (5) above was integrated numerically and is plotted in Fig. 1. is apparent that for any drop s and droplet r to which a value K_1 applies, E' is always somewhat eater than E so that the added turbulence is increasing the chance of collisions; moreover, llisions can occur in cases where $K_1 < 1{\cdot}214$, for which collisions are not possible at all with avity alone.

4. Development of drop-size distribution

The progressive changes in the distribution of droplet sizes with time were calculated using e mean efficiency E' and the method set out by East (1957); the results are shown in Fig. 2. he last curve shown looks very promising as a potential heavy cumulus distribution. The pletion of small droplets is accompanied by an extension of the distribution out to 40 μ, nich could obviously continue with time. The whole family resembles the measured curves Zaitsev (1950) at successive levels in a cumulus. Fig. 2, however, represents the result of an our's action by turbulence and gravity combined. The growing of large cloud drops, which is aly part of the overall transition from young cumulus to raincloud, often takes much less time an this. Still greater speed is required.

In continuing the investigation to greater intensities of turbulence, the effect of gravity is eglected. The random acceleration is assumed to be made up of three independent components hich have Gaussian distributions with equal r.m.s. values. In turbulent motion the three mponents are not independent, and the correlation between them affects the resultant probability stribution. However, both calculations and the experimental results of Panofsky and McCormick 954) indicate that the correlation coefficient is small, even in large eddies; in the smallest eddies, om which come the major contribution to acceleration, the correlation is reduced still further viscous forces. Neglecting correlation, then, the probability distribution of the magnitude accelerations becomes simply a Rayleigh distribution. We assign arbitrarily the value $2g_E$ the r.m.s. linear acceleration in each direction. Whenever the acceleration happens to have is value, the relative velocity between drop s and droplet r is U_2, the relative terminal velocity would have in a steady gravitational field $2g_E$, (provided turbulence has a suitable spectrum). he relative speed $U(t)$ has a Rayleigh distribution with r.m.s. value $3^{\frac{1}{2}}\,U_2$. The mean efficiency is



gure 2. Modification of water-content distribution by coalescence (turbulence with gravity). The curve = 0 is for fair-weather cloud : the curves for later times show the result of one-dimensional turbulence acting with gravity; $(\overline{A^2})^{\frac{1}{2}} = g_E$.

$$E'' = \int_0^\infty kE\,(kK_2)\,P\,(k)\,dk \quad . \quad . \quad . \quad . \quad (8$$

where $K_2 = U_2\tau_r/s$, and

$$P\,(k)\,dk = \left(\frac{2}{\pi}\right)^{\frac{1}{2}} k^2 \exp\{-k^2/2\}\,dk \quad . \quad . \quad . \quad (9$$

The mean efficiency E'' is plotted in Fig. 1; the curve lies above those for E and E' everywhere Moreover, this presentation does not do justice to the enhanced coalescing power of the new turbulence; for a given s and r, the value of K_2 is always nearly twice that of K_1, giving rise t a value of E'' several times greater than E'. For $s = 30$ and $r = 10\,\mu$, for example, $U_1 = 9{\cdot}6$ cm $\sec^{-1}$ and $U_2 = 18{\cdot}6$ cm $\sec^{-1}$. Then $K_1 = 4{\cdot}12$ and $K_2 = 8{\cdot}0$ giving $E' = 0{\cdot}60$ and $E'' = 1{\cdot}10$ the product $U_1 E' = 5{\cdot}8$ cm $\sec^{-1}$ while $U_2 E'' = 20{\cdot}5$ cm $\sec^{-1}$.

The action of this turbulence on the complete water-content distribution (Fig. 3) is t extend it up to 37 μ in 12 min. Evidently an r.m.s. acceleration of $3\frac{1}{2}\,g_E$ would not hasten coales cence sufficiently, but twice this amount would. It is necessary to know what intensity of turbulenc can actually be expected in clouds; some of the evidence will be examined next.

5. Intensity of turbulence

(a) *Theory*

The present-day theory of turbulence arises from the work of G. I. Taylor, Kolmogoroff Heisenberg, Batchelor and others; relevant detail is presented by Batchelor (1953). This theor predicts that when air is set in motion at a very large Reynolds number R, a spectrum of eddie results which extends from the largest eddies that are produced directly by the outside sourc of energy, down to very small eddies which are dissipated by viscosity. The ratio of the larges to the smallest eddy diameters is about $R^{\frac{3}{4}}$; the eddies which lie between the two extremes are said to form the inertial sub-range, and their spectrum (i.e., the curve of energy versus wave number) always follows a universal law. The intensity depends only on the rate at which energy is being absorbed by the turbulence, ϵ.

The Reynolds number R in an active cumulus cloud is of order 10^7 and this is almost certainly sufficient for the inertial sub-range to have the predicted properties; eddy sizes extend from hundreds of metres to roughly 1 mm. Under these conditions a simple formula of Batchelor (1951) is applicable; it is

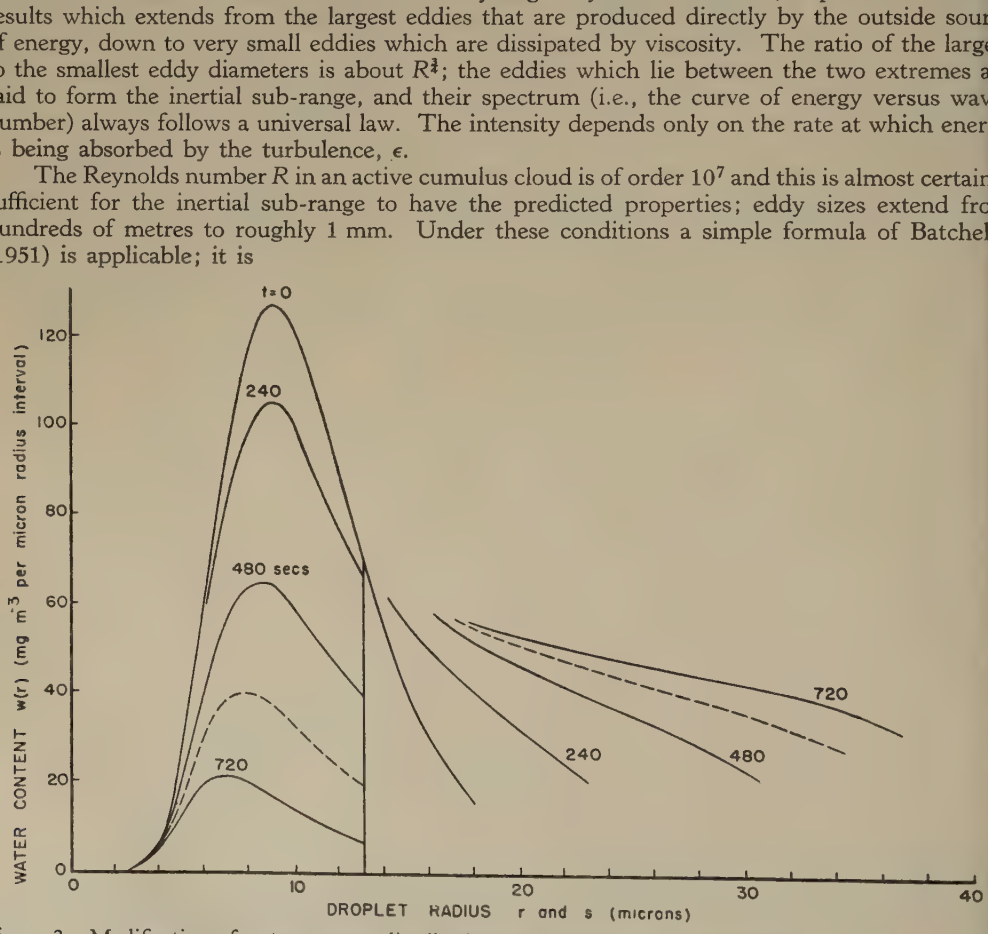


Figure 3. Modification of water-content distribution by coalescence (three-dimensional turbulence). The curve $t = 0$ is for fair-weather cloud : the curves for later times show the effect of turbulent motion; $(\overline{A^2})^{\frac{1}{2}} = 2g_E\,3\frac{1}{2}$. Compare the time scale with that of Fig. 2.

$$\overline{A^2} = \frac{1}{\rho^2}\overline{(\text{grad } p)^2} = 3\nu^{-\frac{1}{2}}\epsilon^{\frac{3}{2}} I \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

ere
$\overline{A^2}$ = mean-square acceleration
ρ = density
p = pressure
ν = kinematic viscosity
ϵ = rate of dissipation of energy by turbulence per unit mass,
d
I = is a number whose value is not known with certainty, but is within a few tenths of 1·3.

tchelor cites experiments by Simmons and Collis which, though very limited in scope, give e formula some verification.

(b) Estimates of intensity from aircraft measurements

To find the acceleration we have to estimate ϵ. It is given by

$$\epsilon = \tfrac{3}{2} A_0 \frac{(\overline{u^2})^{\frac{3}{2}}}{L_p} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

ere
$\overline{u^2}$ = mean-square velocity component in one dimension,
L_p = longitudinal integral scale,
d
A_0 = is a number which, from Batchelor and Townsend's (1948) experiments is about 1·1 (see Batchelor (1953) Fig. 6.1).

Experimental measurements of $\overline{u^2}$ inside clouds are not available, but the results of instrumented aircraft flights have been published by Byers and Braham (1949) and by Malkus (1954). o make use of them we adopt the ' cell ' model used by Byers and Braham and assume :

1. The updraught structure can be identified with the largest eddies.
2. The updraughts account for most of the mean-square velocity.
3. Although the updraught structure is not isotropic, the eddies in the inertial sub-range are isotropic, and the laws of transfer of energy still apply.

Braham (1952) gives results of the Thunderstorm Project. It was found that the median dth for a single updraught in a thunderstorm was 5,000 ft, and the overall width when two draughts were flown through in succession had a median value of 11,000 ft. This suggests periodic updraught structure of ' wavelength ' 6,000 ft or 1830 m, a wave number $_p = 2\pi/1830$ m^{-1} and a longitudinal integral scale $L_p = \frac{3}{4}\pi K_p^{-1} = 686$ m. (See Batchelor 953) p. 105). The median updraught velocity is 22 ft sec^{-1} or 6·7 m sec^{-1}, which is taken to the peak to peak variation in updraught along a line : the mean square vertical velocity, then, $(6{\cdot}7)^2/8$. The quantity $\overline{u^2}$ is defined as the mean square of one of the components in *isotropic* rbulence; since the updraughts are anisotropic, having negligible horizontal components, the ean-square vertical velocity is taken to equal $3\overline{u^2}$ so that $\overline{u^2} = (6{\cdot}7)^2/24 = 1{\cdot}87$ m^2 sec^{-2}. able 1 lists the resulting values of ϵ and $\overline{A^2}$; the turbulence acceleration is quite negligible lative to gravity.

A detailed study by Malkus (1954) of two fair-weather clouds gave updraught velocities about 3 m sec^{-1} and widths of 500 m. Here again the calculated turbulence acceleration is gligible.

ABLE 1. ESTIMATED ROOT-MEAN-SQUARE TURBULENT ACCELERATION FROM AIRBORNE ACCELEROMETER MEASUREMENTS

		$\overline{u^2}$ m^2 sec^{-2}	L_p m	R	ϵ cm^2 sec^{-3}	$\overline{A^2}$ cm^2 sec^{-4}	$(\overline{A^2})^{\frac{1}{2}}/g_E$
aham (Thunderstorm Project)	updraught	1·87	686	$5{\cdot}2 \times 10^7$	61·6	4,450	0·068
	gusts	1·67	197	$1{\cdot}4 \times 10^7$	181	25,900	0·16
alkus (Trade cumulus)		1	375	$2{\cdot}1 \times 10^7$	44	2,690	0·053

Byers and Braham (1949) have published data on gusts. From the description these appear be isotropic eddies derived from the updraughts; only the vertical component is measured (the o horizontal components have properties like the vertical), and upward and downward velocities

are both encountered. Using median values, 3½ gusts per 3,000 ft are encountered, and assumi they act alternately upward and downward, a wavelength of 1,700 ft is obtained, giving $L_p = 197$ r The maximum velocity of each run of 3,000 ft was read, and the median of all runs is 6 ft sec or 1·83 m sec^{-1}, giving $\overline{u^2} = \frac{1}{2}(1{\cdot}83)^2 = 1{\cdot}67$ m^2 sec^{-2}. The acceleration, given in Table 1, still considerably less than gravity.

(c) *Other estimates*

Upper limits can be set to the turbulence intensity in at least three ways. Firstly, the ra of absorption of energy ϵ cannot exceed the available rate of supply from thermodynamic source Braham (1952) estimates that a single thunderstorm cell contains 2×10^{11} kg of air, and th during its lifetime (say 30 min), it is supplied with thermal energy from the latent heat $5{\cdot}3 \times 10^8$ kg of water. The average rate at which energy is supplied, then, is $3{\cdot}3 \times 10^4$ cm^2 sec throughout the cell.

Secondly, the absolute pressure cannot ever become negative anywhere and it does n seem physically likely that it would approach zero closely. Batchelor (1951) showed that f large Reynolds numbers the mean-square pressure fluctuation

$$\overline{p^2} = 0{\cdot}34\,\rho^2\,(\overline{u^2})^2 \quad . \quad . \quad . \quad . \quad . \quad (1$$

For the pressure to approach zero, $\overline{p^2}$ must be comparable with 1 atmosphere^2 or $\overline{u^2}$ wit 17×10^4 (m/sec)^2; all estimates are much smaller than this.

Thirdly, clouds have not been observed to radiate audible sound by virtue of the motio of the air in them. Dr. Heinz Lettau reports (in conversation) from personal experience of fligh in free balloons that it is completely silent even in the heart of a cumulus cloud. Calculation based on Proudman's (1952) equation for the sound power radiated from isotropic turbulen show that at the centre of a turbulent region of radius 1 km the sound would be just audib under ideal conditions if $\overline{u^2} = 10$ (m/sec)^2. Even this consideration, therefore, does nothing t restrict the value of turbulence acceleration.

To sum up, the available experimental data yield values for intensity which are much to small for turbulence to play a significant part in coalescence, but the data are not well suite to this purpose.

Acknowledgments

Thanks are due to Professor J. S. Marshall for his advice and encouragement, and to D G. K. Batchelor and Dr. Heinz Lettau for assistance on various points. The work reported i this Note was sponsored by the Geophysics Research Directorate of the U.S. Air Force Cambrid Research Center.

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Added in proof : Saffman and Turner, in a recent paper on somewhat similar lines, (*J. Fluid Mech.*, **1**, p. 16, 1956) reach a le unfavourable conclusion. The main differences in treatment lie in the values of turbulent intensity and of collision efficiency (the used $\epsilon = 1{,}000$ cm^2 sec^{-3} for cumulus and $E = 1$), and in our neglect of shear as a cause of collisions.

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The forms of ice clouds : II

By F. H. LUDLAM

(Read 17 October 1956. See *Q.J.*, **82**, p. 257)

Dr. R. C. Sutcliffe (*President*) : Mr. Ludlam's observations and photographs illustrate ce more his remarkable talent for picking out significant regularities in the complex detail clouds. His rather speculative theory also seems carefully considered. I should however ve expected to see some visual evidence within the trails if convection currents were in fact ing from them; although the trails may plausibly fall through and warm the air any new rtical instability so produced might be expected to show as a wavy or turbulence structure thin the trail and I could see no evidence of this.

Mr. L. C. W. Bonacina : Cirrus patterns, as viewed from the ground, sometimes appear t and sometimes to bulge out, exhibiting exquisitely beautiful three-dimensional curvature. he flat forms are, I think, most commonly seen rapidly moving from the west ahead of oncoming pressions, the bulging forms in summer anticyclones diversifying the deep blue sky.

Perhaps this difference in the contours of cirrus clouds is mainly a matter of wind shear; ut other factors may be involved at the top of the troposphere and I should like to ask Mr. udlam to explain this characteristic difference on meteorological principles.

Dr. H. L. Penman : Can Mr. Ludlam give a little explanation of the shape of the fallout urves ? Presumably the particles are falling into air layers with different horizontal velocity, nd the terminal velocity is changing as the particles pick up water.

Mr. B. C. V. Oddie : Mr. Ludlam is almost certainly right in thinking that fallstreaks n be formed without a mother-cloud although he has not, in fact, produced any new evidence this point. He has only shown how a cumulus cap can develop over a fallstreak, but that is uite irrelevant to the question of the means by which the fallstreak itself was formed.

Mr. Ludlam's explanation of its formation, which postulates the action of a special and ntirely hypothetical class of freezing nuclei, is not entirely convincing : but it must be admitted at, if one accepts his postulate, it explains the structure shown in his photographs (Plate XII) remarkable detail. If one considers a slowly rising mass or column of air containing these ecial nuclei, it is evident that as soon as its top reaches the level at which these nuclei are ctive, ice crystals will form, and the air will be somewhat warmed and dried. Moreover, the ystals will fall into the lower part of the column, which will also be progressively warmed and ied. The following consequences would then be expected.

) The top of the column would accelerate upwards and it is evident from considerations of continuity that the column must then become narrower just below the top.

) As the heating effect of the crystals progresses down the column, this narrow section will also descend.

) Successively lower slices of the column will be increasingly drier, owing to the action of the falling crystals, and will have to rise to successively higher levels before their own nuclei can form ice crystals. Thus, these crystals will form in a region of relatively high upward velocity where they will, as Mr. Ludlam has shown, be relatively ineffective. One may therefore expect the tapered section, after a little time, to represent the tops of the *clearly* visible fallstreak.

Thus we are led to expect a system consisting of (i) a small cumulus cap, (ii) next, some ry faint trails, (iii), then, the head of the main fallstreak, which should have a tapered, drawn-out pearance, (iv) the body of the fallstreak. We also expect the tapered head of the column to nk gradually, while preserving its drawn-out form. All this is strikingly like what Mr. Ludlam's notographs show.

(*Communicated*) : The fallstreak, if it is one, shown in Plate XIII is obviously a very mature ecimen, and the latent heat released by its formation must long since have been dissipated. We ust seek elsewhere, therefore, for the heat required to produce the little cumuli shown in the notograph. Of course, if the surrounding air is supersaturated with respect to ice, it is only cessary for part of the cloud mass to mix with it in order to release heat, but this original ixing has still to be explained. It might be due to purely dynamic turbulence although the

long straight lines characteristic of these clouds show that the movement is in general perfectly laminar. It is perhaps more likely to be due to the effects of radiation, which render any cloud unstable in time and are of course responsible for the normal degeneration of layer clouds into a regular, cellular cumulus pattern. The regular arrangement of small cumuli of the kind shown in Plate XIII is often very striking : indeed even there the four cumuli shown are quite evenly spaced and the distance between them is apparently something like 2 to 3 times the depth of the cloud layer, as we should expect in a system of Bénard cells. This points to instability within the layer as the triggering mechanism. One may note too that a fallstreak, being newly formed, has not yet developed an impenetrable inversion at its top surface. It is probable that this fact, rather than the latent heat released by its formation, makes the fallstreak particularly liable to generate these little cumuli.

Mr. E. Gold : Mr. Ludlam said the (regeneration) effect would not occur at low temperatures because the available water would be less than 0·1 g/m^3. I suggest that the effect depends on the difference between the dry adiabatic and the saturated adiabatic rate which may be expected to exist at the levels of the fallstreaks. This difference will be smaller at the lower temperature, and a smaller amount of available water, and the consequent rise of temperature, will take the warmed air a greater distance upward.

Mr. R. J. Murgatroyd : Could Mr. Ludlam say if he has any data which suggest that the lapse rate is higher just below the tropopause than it is at lower levels. All the mean curves I have seen based on radiosonde or aircraft measurements show little variation of the lapse rate in the troposphere when the tropopause is low. For high tropopauses the lapse rate decreases considerably in the few thousand feet below the tropopause. There is of course the possibility that the lapse rate is greater when there is cirrus uncinus than at other times but if this is so it must be a very small change which has not yet been detected in the routine aircraft and radiosonde ascents.

Mr. F. H. Ludlam (*in reply*) : In reply to the President's criticism, it can be said that the cirrus trails sometimes *are* distorted beneath the shred clouds, in shapes which indicate the rise of air out of the trails. This is not always, or even often so, and I suggest that the reason is that at the beginning of its motion the air leaves the trail gently, and that it does not attain a substantial upward speed until it is a hundred or two metres higher than the trail and therefore clear of it.

Mr. Oddie correctly points out that I have not produced any evidence to show that fallstreaks form without a mother-cloud. It is difficult to obtain convincing evidence; a well-formed fallstreak develops gradually over a period of half-an-hour or so, and its regeneration or the formation of shred-clouds may begin before the fallstreak is completely grown; indeed, a regeneration process may be necessary if there is to be the persistent crystal formation within a small region required to nourish a fallstreak during such a period. Also, because cirrus clouds commonly move many miles within half-an-hour their aspect changes rapidly, and it is difficult to demonstrate even that the fallstreaks extend downwards. However, at a time when I was disposed to believe that every fallstreak forms by the fall of crystals from a mother-cloud, the most careful search on many occasions failed to show this happening at cirrus levels, and in my first paper (I, 1948) I had to be content with a statement that although ' cirrus cannot always be said to possess a mother-cloud, they may be considered as a type of fallstreak '; I interpreted shred-clouds as the residues of mother-clouds, from which I should have seen the crystal trails descend had I seen the clouds a little sooner. This view is no longer tenable : the shred-clouds are frequently seen to form near mature fallstreaks, but (in the cirrus levels) are practically never seen to form independently and then produce fallstreaks. Moreover, although it is difficult to be sure that a particular fallstreak has not possessed a mother-cloud previous to its observation, I have on a number of occasions observed fallstreaks which did form without any definite mother-cloud.

For these reasons I now believe that it is better to regard fallstreaks as the mothers of the shred-clouds, and that at low temperatures ice-clouds can, and fallstreaks initially do, form without the appearance of any visible mother-cloud. Such a formation would imply the action of crystal nuclei at humidities less than that corresponding to saturation over liquid water – there is nothing hypothetical about such nuclei, for the formation of crystals under these conditions has been observed in the laboratory (I, p. 41).

Mr. Oddie doubts if the cirrus mass shown in Plate XIII should be regarded as a fallstreak. It certainly has not a simple shape, but could nevertheless be composed of several fallstreaks whose lower parts have fused. It almost evaporates in the 50 min covered by the pictures, and I think it may well have previously been in existence for a period less than this. I agree, however, that we should consider processes other than the simple release of ice supersaturation

as the cause of the cumuliform shred-clouds which form above this cirrus mass. The radiative processes which cause convection in cloud layers below the cirrus levels cool the cloud tops, and therefore do not produce towers which rise to greater heights. At the cirrus levels, conditions may be different, and it would be interesting to see them examined quantitatively. Instability in a shallow layer including the cloud tops could also be produced if the entire cloud mass rose by a few hundred metres during its passage over a ridge, a not unlikely event. I quite agree that a strong lapse rate above the clouds is an important condition favouring the formation of the shred clouds, but would not seek any close relation between the air movement they represent and that in Bénard cells.

In answer to Mr. Gold, the distance which buoyant air may rise depends upon its original excess temperature and the outside lapse rate; the value of the former when produced by the release of ice supersaturation is very small at low temperatures, whereas the outside lapse rate, as Mr. Murgatroyd points out, is not usually as great as the saturated adiabatic. Consequently at these temperatures the maximum likely rise of the warmed air is usually only some tens of metres. Mr. Bonacina's remarks correctly imply that there are two distinct kinds of cirrus : the fallstreak forms, which have a characteristic vertical extent of a kilometre or two, and the shallow, 'flat,' forms, which are only a few hundred metres thick, I think it is true that the former predominate in summery weather, for then the middle and upper troposphere are frequently unstable, and fallstreak regeneration, even with castellanus formation, is favoured. On the other hand the rapid and widespread formation of cirrus near cyclones ensures the production of extensive layers containing relatively abundant and therefore small crystals which have rather small fall-speeds, so that the dense and deep fallstreak is not the characteristic form. The strong wind shears and the marked stability in these situations also inhibit the development of fallstreaks.

I would refer Dr. Penman to my first paper (I, 1948, p. 48) in which there is some tentative discussion of the shapes of fallstreaks.

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Space and time variations of charged nuclei and electrical conductivity of the atmosphere

By RITA C. SAGALYN and G. A. FAUCHER

(Read 17 October 1956. See *Q.J.*, **82**, p. 428)

Dr. B. J. Mason : I welcome this paper. By making their measurements well above the surface, where local sources of nuclei and other local effects are usually complicating factors, the authors have, I believe for the first time, clearly demonstrated the importance of coagulation in determining the nucleus concentration and the electrical conductivity. The observed decay of nucleus concentration between 1500 hr and sunrise is immediately explicable in terms of coagulation when one recalls that the half-life of an aerosol containing 10^4 particles/cm^3 of radius 10^{-6} cm is 15 hr. I am not so sure, however, that the rather sudden increase in the nucleus population which occurs at sunrise, and which is also observed at the surface, can be explained entirely in terms of transport from below by turbulent and convective mixing. I am inclined to think that there must be a marked increase in nucleus production, perhaps by photo-chemical reactions. In this connection it would be interesting to make nucleus counts around sunrise over the sea, both at the surface and aloft, and in remote land areas relatively free from combustion products.

Mr. R. J. Murgatroyd : The number of the large ions and their distribution in the vertical are very similar to those of Aitken nuclei obtained in flights by Mr. G. J. Day over this country. The scope of aircraft research in nucleus investigation is now very large and has been extended in recent years to dust sampling, giant salt nuclei, spores and freezing nuclei, in addition to those discussed above. In general it seems they all exhibit rather similar patterns as regards vertical distribution, with the exception of freezing nuclei. The depletion of concentration above inversions suggests that in rain-making experiments depending on the release of silver iodide from the ground there is very little chance of the particles reaching sufficiently high levels if any stable layer exists. For this and other reasons it would be very interesting to see this work extended to cloudy and rainy conditions.

Another desirable improvement in nucleus sampling in general is the production of a device which will give satisfactory counts down to 1 or even 10 per cm^3. If this could be done it would be relatively easy to obtain nucleus counts in the upper troposphere and lower stratosphere where they are of considerable interest.

Dr. F. J. Scrase : I would like to ask whether Smoluchowski's formula for the rate of coagulation applies only to uncharged particles. If so, to what extent would the calculated rate of coagulation be affected by the fact that large proportions of the nucleus concentration carry positive and negative charges ?

Dr. T. W. Wormell : Among many things which I have learnt from this paper is the convincing demonstration that the decrease in nuclear concentration which occurs typically at many levels during the night is due primarily to coagulation. This is presumably an irreversible change ; the small nuclei are permanently lost. The increase in numbers, which is observed during the day, would appear therefore to involve not only more active turbulent mixing but, also, a copious supply of new small nuclei in the lower layers from sunrise onwards.

I have two questions; first, what is known as to the source of these new nuclei ? Are they due to combustion or other human activities or are they produced equally in sparsely inhabited regions ? Secondly, the discussion of the decrease in total nucleus concentration is based on observations of charged nuclei and a theory of the equilibrium between charged and uncharged nuclei. Is the approach to equilibrium sufficiently rapid for an equilibrium theory to be an adequate guide to the interpretation of the observations when all concentrations are varying with time ?

Mr. F. H. Ludlam : The distribution of nuclei vertically by convection was much in evidence during summer field work in Central Sweden. There the decrease from morning to afternoon of the giant hygroscopic nuclei counted at the ground was a regular occurrence in winds with a westerly component, and clearly due to the overland convection reaching much higher levels than the stirring motions which carried the nuclei into the atmosphere over their source, the ocean.

Dr. R. C. Sutcliffe (*President*) : I am pleased that the question of the source of the nuclei has been raised. While the authors' mechanism for reducing the numbers is important it cannot alone provide for a diurnal variation : this requires a diurnal variation in production rate which must be rather general within the air mass if the variation occurs generally. The fact that the curve follows rather closely the curve of diurnal surface temperature variation suggests one should look for a surface source linked with convection from the surface, possibly with evaporation or evapotranspiration.

Dr. R. S. Scorer : In attributing the lowering of the level of high concentration during the night to subsidence is it implied that this subsidence occurs only at night or that it is obscured by other things by day ?

Dr. R. C. Sagalyn (*in reply, on behalf of both authors*) : In reply to Dr. Mason, concerning the explanation of the observed increase in nucleus concentration beginning approximately at sunrise, our results indicate quite clearly that in the altitude region 700 ft to the top of the mixing level, the principle cause for increasing concentration is nucleus transport from the lower levels. The increase does not occur simultaneously at all levels and the maximum height to which the nucleus increase extends coincides with the maximum height to which a change to a more unstable temperature gradient is observed. In more recent measurements over the Atlantic Ocean we observe no increase of nucleus concentration in the altitude range 500 to 20,000 ft following sunrise. This does not exclude the possible role of photo-chemical reactions in particle formation very near the surface.

In answer to Dr. Scrase's question, Smoluchowski's theory was derived for all spherical particles in a given volume whose movement is determined by Brownian motion until they approach within a distance R of another particle. R is considered to be the radius of an attractive sphere about the particle of interest. Smoluchowski took this radius to be the radius of the nucleus. Physically, of course in the case of charged particles the influence of the electric field must be added to the Brownian movement. However, simple calculation shows that the radius at which the electrostatic force for singly charged particles is equal to the force due to Brownian motion is approximately equal to the radius of the nucleus. Brownian motion is thus the dominant influence determining collisions. This is different from the case of collisions between small ions or between small ions and nuclei, where the effect of electrostatic attraction on the collision frequency cannot be neglected. Our measurements in normal atmospheric air indicate that the number of multiply charged nuclei is negligible.

Dr. Wormell has raised a very pertinent question concerning the use of a theoretical value for the ratio of the number of charged to uncharged nuclei. Whether or not it is valid to use the theoretical expression for the ratio of charged to uncharged nuclei, which assumes equilibrium to obtain the total nucleus concentration from measurements of charged nuclei, depends on the value of the term representing the change of nucleus concentration per unit time compared to the other terms in the equilibrium relation. Comparison of the measured rate of change of charged nuclei with the other terms in the ion balance relation computed assuming Bricard's theoretical expression for the attachment coefficients (which do not depend on the existence of equilibrium) and measured values of small ions and charged nuclei indicates this method is valid. For example, for a measured positively charged nucleus concentration of 2,000 per cm^3, dN/dt was found to be less than one hundredth of any of the collision frequency terms. All calculations take into account the change of nucleus concentration with time.

It would be more desirable to compare Smoluchowski's theory with measurements of total nucleus concentration as a function of time. We hope to do this in future experiments.

These experiments indicate that the nuclei we are considering are principally due to combustion and other human activities. Over the relatively uninhabited mountainous regions of Southern California and over the Atlantic Ocean the nucleus concentration was consistently found to be lower than over the eastern part of the United States, with the exception of regions of forest fires when the nucleus concentration was found to increase.

In agreement with Dr. Sutcliffe's remarks we have shown that transport of nuclei from lower levels, by turbulent and convective mixing, and nucleus coagulation are the dominant influences determining the diurnal variation above 700 ft in fair weather. The rate of production of nuclei at or near the surface will certainly modify the diurnal variation during the day and as pointed out in this paper (*Q.J.*, **82**, p. 444) often becomes the dominant mechanism at the surface in the middle of the day. The results do not require a diurnal variation in the rate of production of nuclei but only in the rate of vertical transport, linked to the daily variation of surface temperature as pointed out by Dr. Sutcliffe.

Since we have measurements of both the vertical profile of nuclei and its variation with time, the flux of nuclei as a function of height can easily be computed. Additional information can be obtained about the source and the mechanism of transport by solving the continuity equation for the Austausch coefficient as a function of height. The effect of coagulation on the concentration must be included in the calculations. This will give the average Austauch coefficient over a 25-mi path as a function of height and time. Further, information about the nature of the source and its variations could best be obtained by making measurements in the first 1,000 ft above the surface.

It is important to point out that for the time-variation experiments, in order to minimize the effects of single sources, a fixed flight path was chosen, approximately 25 mi long, over which there could be found no large isolated source of industrial pollution. Over large cities and over isolated smoke stacks increases in nucleus concentration and a decrease in conductivity are easily observed in the lowest 1,500 ft above the surface.

Replying to Dr. Scorer, these results imply that on fair-weather days subsidence is often obscured during the day by turbulent and convective mixing in the lowest 10,000 ft above the surface.

Proceedings at meetings of the Society

Ordinary Meeting, 17 October 1956

The following candidates were declared to have been elected by the Council during the summer recess :

Fellows

Bath, Allen Tristram, B.Sc., Bureau of Meteorology, Brisbane, Queensland, Australia.
Doherty, George, 31 Braehead Street, Kirkintilloch, Glasgow, Scotland.
Drinkwater, Peter Joseph, 151 Christchurch Avenue, Kenton, Middx.
Duncan, Walter, A.C.I.S., 'The Cottage,' Clashnoir, Braes of Glenlivet, By Ballindalloch, Banffshire, Scotland.
Evans, Donald Phillip, 59 Park Avenue, Rushden, Northants.
Flegg, Flt. Lt. H. G., M.A., Officers' Mess, R.A.F. Spitalgate, Grantham, Lincs.
Frettsome, Eileen M., B.Sc., 29 Orpin Road, South Merstham, Surrey (*transfer from Student Membership*).
Houseman, John, 55 The Boundary, Goldington, Bedford, Beds.
Macdonald, Adam, 10 Gordon Terrace, Bettyhill, By Thurso, Caithness, Scotland.
McKinna, Eric Charles, B.Sc., 47 Heywood Street, Mt. Gravatt, Brisbane, Queensland, Australia.
Nunn, Martin William, B.A., 3 Stephen Road, Barnehurst, Bexleyheath, Kent (*transfer from Student Membership*).
Orr, Sqn. Ldr. Norman William, Officers' Mess, R.A.F. Bassingbourne, Royston, Herts.
Salter, Wallace, B.Sc., 'Virley,' 157 Stanley Road, Hinckley, Leics.
Smith, Roy Henry, Edward A. Pearce and Partners, 28 Baker Street, London, W.1.
White, Gordon, 87 Greyhound Lane, Streatham, London, S.W.16.

Foreign Members

Butto, Matti, Meteorological Office, Baghdad Airport, Baghdad, Iraq.
Mintz, Professor Yale, Department of Meteorology, University of California, Los Angeles 24, California, U.S.A.
Phillips, Dr. Norman A., 9 Scotland Road, Lexington, Massachusetts, U.S.A.
Shourcatly, Mohamed Abdo, Meteorological Office, Mazzeh Airport, Damascus, Syria.
Stadsklev, Glenn Harris, Lt., U.S.A.F., 755 West Union Street, Roseburg, Oregon, U.S.A.

Associate Member

Boyd, Trevior Robert, 36 Charlotte Street, Chermside N.4, Brisbane, Queensland, Australia.

Student Members

Bell, Clive, 17 Dorset Crescent, Billingham-on-Tees, Co. Durham.
Maddever, Richard Sheeres, South Petherwin, Launceston, Cornwall.

The following candidates were then declared to have been elected that day by the Council :

Fellows

Barratt, Peter, B.A., 77 Onslow Road, Richmond, Surrey.
Booth, Robert Ernest, 16 Pamela Gardens, Eastcote Park, Pinner, Middx.
Broadhurst, Captain L., 8 St. Anthony's Road, Bournemouth, Hants.
Kerr, Alan Ripley, 91 Beaufort Mansions, London, S.W.3.
Launey, George L., D.F.C., 4469 Earnscliffe, Apt. 4, Montreal, Canada.
Shaw, Elizabeth Mary, B.Sc., Bedford College, Regent's Park, London, N.W.1.
Smith, Walter Verdun, 7 Mirfield Grove, Blackpool, Lancs.

Foreign Member

Greene, Clark Doane, S.M., Rock Hall, Maryland, U.S.A.

Student Members

Arnott, Jennifer, 3 Hutton Lane, Guisborough, Yorks.
Baston, Neville, 3 Chantry Place, Colyton, Devon.
Baylis, Michael Henry John, 42 Dennis Road, Weymouth, Dorset.
Coombs, Margaret A., 49 Barbourne Road, Worcester.
Garrett, Marcus Colin Henry, 39 Wychwood Avenue, Luton, Beds.
Graeme, Anthony Francis, 1 Station Road, Bosham, Sussex.
Grainger, Maureen E., 1 St. Fagan's Road, Cardiff, South Wales.
Grebby, Christine, 19 Vale Road, Southborough, Tunbridge Wells, Kent.
Harrold, Terence William, 18 Garden Road, Sheringham, Norfolk.
Haynes, Diana E., 30 Summerleaze Park, Yeovil, Somerset.
James, Julia Mary, Rookery Nook, Wellington Heath, Nr. Ledbury, Herefordshire.
Langridge, Robert, B.Sc., 1 St. Joseph's Cottages, Upper Froyle, Nr. Alton, Hants.
Oxley, Wilfred John, 119 Calder Road, Maidstone, Kent.
Slow, Hilary, 1 Tower Road, Barbourne, Worcester.
Spalding, Terence Rodney, 37 London Road, Ipswich, Suffolk.
Spencer, Peter Murray, 22 The Mount, Reading, Berks.

The following papers were read and discussed :

(1) ' The forms of ice clouds : II ' by F. H. Ludlam (*Q.J.*, **82**, p. 257).

(2) ' Space and time variations of nuclei and electrical conductivity of the atmosphere ' by Rita C. Sagalyn and G. A. Faucher (*Q.J.*, **82**, p. 428).

Ordinary Meeting, 21 November 1956

The following candidates were declared to have been elected that day by the Council :

Fellows

Anson, William Weatherburn, B.Sc., ' Riverside,' Lintzford Road, Rowlands Gill, Co. Durham.
Archer, William Sidney, Forge Cottage, Kirdford, Sussex.
Croom, David Lindsay, B.Sc., 103 Baldock Road, Letchworth, Herts.
Findlater, John, 30 St. John's Road, Watford, Herts.
Holmes, James, 46 Bellevue Road, Prestwick, Ayrshire, Scotland.
Large, Michael Ison, B.A., West Wing, Kermincham Lodge, Holmes Chapel, Crewe, Cheshire.
Malan, David Johannes, Institute of Geophysical Research, University of Witwatersrand, Johannesburg, S. Africa.
Rohan, Patrick Kilian, M.A., 3 Laurel Villas, S. C. Road, Limerick, Ireland.
Scott, Alan James Steward, 124 Seely Road, London, S.W.17.
Shaw, Raymond, B.Sc., 46 Egerton Gardens, London, S.W.3.
Wooldridge, George Charles, 88 Coventry Road, Market Harborough, Leics. (*reinstatement*; *resigned* 1950).

Foreign Members

Jordan, Charles L., National Hurricane Research Project, P.O. Box 271, West Palm Beach, Florida, U.S.A.
Rebman, Edwin J., 1860 East Alder Street, Walla Walla, Washington State, U.S.A.
Suzuki, Seitaro, Prof., No. 915 Toyoshiki, Kashiwa-shi, Chiba Prefecture, Japan.

Associate Members

Duquet, Robert Theodore, M.A., 73 West 174 Street, Bronx 53, New York. U.S.A.
Wright, Donald Sidney, B.Sc., 8 Kathleen Street, Pascoe Vale South, Victoria, Australia.

Reviews

Beiträge zur Physik der Atmosphäre. Bd. 29, H.I. Edited by H. Flohn, B. Haurwitz and J. Van Mieghem. Frankfurt a.M. (Akademische Verlag), 1956. 55 D.M. per vol. of 4 Pts.

In 1945 there appeared one part of volume 28 of *Beiträge zur Physik der freien Atmosphäre*, an international journal of high reputation, founded by Assmann and Hergesell in the early days of the exploration of the upper air, published in Leipzig. For long it seemed that it was to be the last.

The publication which we now notice, though its title is a little different, and its place of origin changed, has the inscription ' 29 Band, Heft I,' and is clearly intended to carry on the tradition of its famous predecessor. It contains a short biography of Assmann by H. Koschmieder and papers on lee-waves by J. Zierep, on a dust-storm in Iraq by Abdul Jabbar Abdullah, and on experiments in convection by H. v. Tippelskirch. Manuscripts in German, French, and English are invited. We wish the revival every success.

G.D.R.

Journal of fluid mechanics, Vol. I, Pt. I. Edited by G. K. Batchelor. London (Taylor & Francis), May 1956. £5.10.0. per vol. of 6 Pts.

This new journal, in the publishers' words ' exists for the publication of theoretical and experimental investigations of all aspects of the mechanics of fluids.' Much of physical and dynamical meteorology is covered by this description, and of eight papers in the first number, three (' On the collision of drops in turbulent clouds ' by P. G. Saffmann and J. S. Turner, ' Long waves on a rotating earth in the presence of a semi-infinite barrier ' by J. Crease, and ' Vibrational relaxation in oxygen and nitrogen ' by V. Blackman) are of direct interest to meteorologists, who clearly cannot afford to neglect the new journal. But we may hope also that workers in other fields of aero- and hydrodynamics will be brought to a knowledge of, and an interest in, meteorological problems through this journal, and that in this way it will render a double service to our science.

G.D.R.

Physikalisch-Statistische Regeln als Grundlagen für Wetter und Witterungsvorhersagen, Part I. By Dr. F. Baur. Frankfurt a.M. (Akademische Verlag), 1956. Pp. viii, 138; 30 Figs.; 76 Tables. 42 D.M.

In his preface, Dr. Baur says that his aim in this book is to show that, guided by physical considerations, rules can be found regarding connections between general weather patterns and solar phenomena by statistical investigations based on a large body of observations. Statistical investigations are also included which concern not the general weather pattern but the daily weather. These are intended to show that the use of statistics based on physical considerations provides a better base for short-range forecasting than the use of typical examples.

The book is in four sections of which the first deals with statistical bases for forecasts of the 24-hr pressure change at Potsdam and for forecasts of short spells of wet or dry weather in middle latitudes. The next section deals with theoretical considerations into variations of the general atmospheric circulation as guides for statistical research on ' Grosswetter.' This is illustrated by eight typical patterns of upper tropospheric airflow and the corresponding surface-pressure distributions.

The third section is mainly observational, giving figures for the zonal circulation in middle latitudes for each day of the years 1949 to 1951. The last section is mainly statistical, and deals with connections between large-scale weather and sunspot numbers.

The book is not without merits : it is excellently produced and contains a large amount of observational material which may be of use to other workers. Moreover, there is probably an element of truth in many of the results. However, it should be judged as a contribution to knowledge in 1956. The importance of applying statistical methods based on physical considerations to meteorological problems is already widely understood and the second and third sections do not call for comment. The important parts are the first and last, in which Dr. Baur seeks to apply his principles in practice. These consists of statistical analyses of various bodies of data, on which are based rules for forecasting.

The standard of the statistical analysis may be judged from an example. The author gives a contingency table showing the 24-hr pressure change at Ponta Delgada on each summer day during the decade 1931-40 plotted against the 24-hr pressure change at Potsdam during the next day. These variables show a correlation of $-0{\cdot}17$, which with 620 observations is claimed to be very significant. Certainly the value is statistically significant provided that (1) both variables are normally distributed, (2) the data are not serially correlated and (3) the value $-0{\cdot}17$ has not beeen quoted because it is the highest of a group of correlation coefficients between pressure changes at Potsdam and those elsewhere. In fact the first two conditions are unlikely to be fulfilled, while Dr. Baur tells us nothing about the third. In any case, even if statistical significance is granted, the proper deduction from a correlation of $-0{\cdot}17$ is surely that no worthwhile physical connection has been demonstrated. The author does not make this deduction, but proceeds to a very elaborate multiple contingency table in which 1,440 Potsdam pressure changes are sorted into 432 categories depending on the values of 6 independent variables. This table is made the basis of ten ' rules for forecasting ' referring to the categories in which the observations show most agreement among themselves. This method is capable of producing rules from entirely unrelated data.

As an example of the ' rules,' one may be quoted (taken from a different part of the work) :

Rule 15 : Given the following conditions during the summer half-year in the middle latitudes of the northern hemisphere on the continents in areas which are not too markedly affected by orographic disturbances:

(a) a drop in the daily mean temperature of at least 4°C is followed for two successive days by a simultaneous rise of the daily mean temperature and of the atmospheric pressure, from evening to evening, and

(b) the pressure rise is concurrent with overcast skies for at least 12 hours in succession.

We can expect with a high degree of probability that mainly dry weather will prevail during the following 5 days, provided that the temperature lies below its normal value on the last day of the temperature drop, that the pressure, reduced to sea level, amounts to at least 1020·0 mb on the second evening of the simultaneous pressure and temperature rise, and that no sharp pressure changes of either sign have occurred immediately before these last two days.

These examples of the method and the results should enable the reader to decide how far the work will repay his attention.

J.M.C.

Handbuch der Geophysik, Bd. VIII : *Physik der Atmosphäre I*, Chapters 12, 13, 14. Edited by F. Linke and F. Möller. Berlin (Gebrüder Borntraeger), 1955-56. Pp. 174; 65 Figs.; 31 Tables, 1 colour plate. Np.

These three chapters of the *Handbuch* form a coherent group describing all the physical processes of atmospheric refraction, reflection, scattering and absorption in the optical region of the spectrum. With the exception of a brief mention of the ' green flash ' by R. Meyer in Chapter 13 and passing reference to the colour of the sky and the ' purple light ' by F. Volz in Chapter 14, the chapters are almost entirely concerned with principles and it is to be assumed that there will be further detailed chapters, of similar standard, on the actual atmospheric optical phenomena. If this is so, then this new edition of the *Handbuch* will be a document of exceptional value, for this particular sample is exhaustive, systematic and (as far as the reviewer's German allows him to judge) well written.

Chapter 12 is perhaps the least important of the three. F. Lauscher writes on ' Optik der Gewässer ' and treats, very fully, problems of refraction and reflection at flat, curved, and wavy water surfaces, and also the complicated question of internal scattering in water. Brief mention is made of instruments for measuring in natural waters and some results are given. This is a rather typical geophysical topic. The theory is excellent as far as it goes, but when faced with the results of field measurements one finds that it does not go very far and the treatment becomes empirical.

Chapter 13 is a long chapter (52 pp.) by R. Meyer, entitled ' Atmosphärische Strahlenbrechung ' which deals essentially with three problems : refraction of stellar images by a stratified atmosphere with a uniform temperature gradient (the problem of accurate astronomical positional measurements); effects of irregular density changes (the problem of astronomical ' seeing '); the height of the horizon and mirages. This concentration on the optical region means that only refractive index changes due to temperature changes are considered. An opportunity is certainly missed of presenting a unified treatment embracing the effect of water vapour on refraction of very high frequency radio and the effect of the ionosphere on medium and low frequency radio. These problems are entirely analogous and, for example, the recent resurgence of interest in problems of atmospheric ' seeing ' is, in this country at any rate, directly connected with the development of ionospheric measurements on winds and turbulence.

Chapter 14 is by F. Volz and is entitled ' Optik der Tropfen; Abschnitt I : Optik des Dunstes.' The whole section on optics of droplets is going to be a remarkable compilation, for this part alone contains 75 pp. and has about 160 references. As a sign of the extent to which this small part of physical meteorology has grown, it may be pointed out that Dr. Volz scarcely touches on the theory of scattering and goes almost straight to the results, missing out all the work on this theory beautifully presented in his Doctoral Thesis by Van der Hulst. From the point of view of a specialist this is perhaps a pity, but Dr. Volz is probably quite correct in his choice of presentation. All the references are given and the descriptive matter is easily readable. Further excursions into theory would have resulted in a most unwieldy article.

Apart from the lack of detailed theoretical treatment Dr. Volz's article embraces the subject in all its aspects both theoretical and observational. Considerable use has to be made of diagrams which are complicated and at first sight confusing, although careful study shows that their meaning is in fact quite clear. It is a very surprising thing that publishers of the reputation of Gebrüder Borntraeger should be content with a wide variety of printing in the diagrams, and including some examples of most unprofessional free-hand.

All criticisms however are unimportant when compared with the completeness and authoritative nature of the articles. The completed volumes will undoubtedly be a necessity for all institutions and many research workers.

R.M.G.

The sun. The solar system, Volume 1. Edited by Gerard P. Kuiper. University of Chicago Press, 1953. Pp. xix, 745. $12.50 (Cambridge University Press, 94s.).

This book on the sun is the first result of the great enterprise of Professor Kuiper and the Chicago University Press to give a systematic and comprehensive account of the solar system. The series of 4 volumes

is to be written by 56 authors from 10 countries and the first volume on the sun, which was published in 1953 contains 9 of the proposed 51 chapters. Perhaps the most surprising aspect of this book is that over twenty distinguished authorities from widely separated institutions have produced a detailed survey of the sun which is pleasing both in its continuity and in its readability. The Introduction by Leo Goldberg is followed by a description of the sun as a star in which Bengt Strömgren deals with the physics and chemistry of the interior These prepare the way for three remarkable chapters by M. Minnaert on the photosphere, H. C. van de Hulst on the chromosphere and the corona and by K. O. Kiepenheuer on solar activity, each of which is so complete and authoritative as to provide a satisfying book in its own right. These are interspersed by a short chapter on the identification of solar lines by Charlotte E. Moore, and are followed by J. L. Pawsey and S. F. Smerd on solar radio emissions, a fine contribution which deals completely and elegantly with the outstanding developments in this work in the twelve years since the discovery of the phenomena of radio emissions from the sun. T. G. Cowling in his chapter on Solar electrodynamics is faced with the difficult task of presenting the rival theories of interpretation of solar phenomena. The final chapter of 120 pp. is comprised of 13 sections by various authors on empirical problems and equipment. These include a section on the techniques of radio-frequency observations as well as several on the conventional methods of solar observation.

Kiepenheuer's chapter contains a short section on solar-terrestrial relationships as exemplified by the relation of corpuscular streams to geomagnetic storms and aurorae, otherwise the entire volume is essentially concerned with the problems of the sun itself. Nevertheless if any justification for the meteorologists' concern with this book is needed it is provided by Strömgren who concludes his chapter on the interior of the sun by raising the question of the solar variations which might be important in studies of the climatology and geology of the earth.

A.C.B.L.

Hurricanes. By I. R. Tannehill. London (Geoffrey Cumberlege), Ninth Edition, 1956. Pp. x, 308; 141 Figs.; Tables. 36s.

The many meteorologists, professional and amateur, who regard I. R. Tannehill with admiration and gratitude for his classic work on West Indian hurricanes will be glad to know that the author is not allowing this book to become out-of-date. The ninth edition contains an additional five pages at the end of the Appendix which describe, in continuation of the practice carried out for previous years, the hurricanes that were reported during the years 1952 to 1955. It seems unfortunate that the publishers should slightly spoil a valuable book by relegating the description of hurricanes which occurred during the years 1945 to 1955 to the end of the Appendix, which gives an account of recent practice and experience in observing and forecasting hurricanes, instead of adding them to the end of Chapter 14, ' Hurricanes of the Twentieth Century.'

C.R.B.

200 miles up : the conquest of the upper air. By J. Gordon Vaeth. New York (Ronald Press). 2nd Edition, 1955. Pp. xiii, 261; 39 Figs.; Plates. $5.00.

The author tries to tell, for a general audience, the story of high-altitude exploration up to the startling announcement by the United States that they hope to launch an artificial satellite during the International Geophysical Year. To write such a book is a task that one might envy, especially if, like Mr. Vaeth, one has an official position in the Office of Naval Research which gives access to many first-class photographs and technical details of great interest. The book can be recommended for the figures, plates and technical details, but the main body of the text will not please the average reader.

Mr. Vaeth makes two major errors of judgment. Firstly, he is clearly not familiar with the details of the more scientific sides of the work, but he tries to bluff his way through. As a result there are a number of errors and there are statements which are so fumbling that they convey little except that the writer is in difficulties. Secondly, he believes that a racy journalistic style is the best means of popular presentation and he also believes that he can handle this style. This style of writing, however, requires an unusual gift if it is to be successful. The author does not have this particular gift and the result will grate on the nerves of the least sensitive reader.

The first chapter of importance reviews the properties of the upper atmosphere. The title of this chapter is ' The atmospheric ocean,' illustrating typically how the author can confuse the inexpert reader by introducing irrelevant notions. The chapter contains many errors and inaccuracies. On p. 21 the author neglects to include water vapour in his list of atmospheric constituents, and makes the completely unfounded assertion that above 30 miles at latitude 45° the oxygen content of air may fall from 21 per cent to 18 per cent. On p. 24 he makes the extraordinary statement that the blue colour of the sky is due to dust scattering. On p. 26 when listing the airglow emissions he omits the hydroxyl emissions which are responsible for the greater part of the energy.

In the next chapter on ' Upper-air research instruments ' the author is hesitant, confusing and often inaccurate, but in the subsequent chapters he seems more at home and the quality of the book improves. There are general chapters on high-altitude vehicles, the principles of rocket flight and the White Sands proving ground, and also detailed chapters on the Skyhook balloon, the V2, the Aerobee, the Viking and

he artificial satellite. Much of the information in these later chapters is certainly not available elsewhere nd apart from the fact that they are sometimes repetitive and, on occasions, introduce what seems to be rrelevant detail, these chapters would be valuable but for the unfortunate style and composition. To make t clear to the reader what he will have to put up with, a few of Mr. Vaeth's sayings are given below without comment.

Ozone : Ozone is oxygen which has been changed from O_2 to O_3, (p. 11). *The atom* : The atom itself s so small that it has never been seen by man, (p. 48). *The Geiger counter* : Essentially, it is an electrified gas-filled tube, (p. 49). *The ionosphere* : A vast vacuum-like area, the ionosphere is a region filled with electrical phenomena of the greatest importance to science and particularly to radio communications, (p. 11). *Research workers* : To some of them, they (*cosmic rays*) have brought disaster; to others the Nobel Prize (p. 47). *Rockets* : High speed is characteristic of rockets, and the Aerobee is no exception (p. 183).

To sum up. This is a case of excellent material spoilt by bad writing. It can only be hoped that other workers in the field will not hesitate to try to improve on it.

R.M.G.

Thermodynamic tables and other data. By R. W. Haywood. Cambridge University Press, 1956. Pp. 23; 12 Tables. 2s. 6d.

These are tables for the engineer, all quantities being expressed in engineering units. They include tabulated values for the specific volume, internal energy, enthalpy and entropy of water and steam in equilibrium over a wide range of pressures and corresponding temperatures, similar data for some common refrigerants, and for superheated steam for temperatures between 150°F and 1,600°F. They will be of little use to the meteorologist.

B.J.M.

Corrigendum to April 1956 issue

C. H. B. Priestley—

p. 171, Eq. (15)—

for $$-\frac{dw}{dz} = k - \frac{g}{\theta_e}\theta'$$

read $$-\frac{dw}{dz} = k - \frac{g\theta'}{\theta_e w}$$

Obituary Notice

Mr. S. E. ASHMORE, B.Sc., A.Inst.P.

By the death at the War Memorial Hospital, Wrexham, on 14 October 1956 of Sydney Eric Ashmore the Society lost a recent Darton prize-winner, a former member of Council (1945-46), and an assiduous contributor to this *Journal* and to the magazine *Weather*. His fatal illness, an obscure fever, overtook him a few days after he had attended the Society's first meeting in Wales – at Aberystwyth on 26-28 July, when he spoke by invitation on Welsh snowfall. Though an avowed Welshman with all the national pride of the race, Ashmore was a Londoner by birth. Born at Edmonton on 1 September 1909 and educated at Churcher's College, Petersfield, and University College, Southampton, he took the London B.Sc. degree in 1931 and then held teaching posts in Birmingham from 1932 to 1936. On 1 January 1937 he was appointed physics master at the well-known Grove Park School, Wrexham. There he passed the rest of his life, achieving high popularity among boys and staff alike. 'He was a friend to his pupils and held in affectionate regard by his colleagues. He gave of himself to his school . . . he lived for it . . . an original teacher, and a very great teacher.' These words formed part of the tribute spoken by Ashmore's Head Master, Mr. E. Haddon Roberts, O.B.E., M.A., M.Sc., at the memorial service for him in Wrexham.

Meteorology made its first call to Ashmore at a very early age, as he afterwards showed by citing thunderstorm observations which he had recorded before he was 12 years old. At 18 he started his long series of letters and notes to the *Meteorological Magazine*. Atmospheric optical phenomena claimed his chief attention at that time. Soon his interest in weather science became more catholic, and after his election to Fellowship in this Society, on 15 June 1932, his communications to the serial publications covered a wide range of subjects. In all, he contributed about a dozen papers and notes to the *Quarterly Journal* and some 35 articles and other items to the magazine *Weather*. Outstanding among these are 'The diurnal range of temperature and the geographical distribution of its annual variation' (*Q.J.*, **65**, p. 554), setting forth the results of a massive piece of harmonic analysis; 'The rainfall of the Wrexham district' (*Q.J.*, **70**, p. 241), a model study of its kind; 'Records of snowfall in Britain' (*Q.J.*, **78**, p. 629), presenting equations to improve the intercomparability of data supplied by observers with varying station-exposures and opportunities for vigilance; 'A North Wales road-mirage' (*Weather*, **10**, p. 336), comprising a mathematical discussion of an organized investigation which gained the author one of the Darton Prizes for 1955.

On joining the staff at Grove Park, Ashmore gave prominence to meteorology in the physics curriculum from the first. After a few years he arranged for the establishment of a well-equipped climatological station in the school grounds so as to let his pupils acquire practical experience of standard observational routine. So successfully did he infuse his own enthusiasm for the work into the boys and his fellow masters that the daily readings have always been as regularly and carefully taken during holidays as during term time. Later he founded a school meteorological society and was able, after much effort, to fulfil an early ambition by setting up and maintaining a mountain climatological station on his own account – at Bwlchgwyn, 1267 ft above m.s.l. on the Denbighshire heights. He named it the Charlotte Lyndon Weather Station in memory of a former Fellow of the Society who had greatly befriended him in his youth and who communicated the first of his papers to be published in this *Journal*. Both Grove Park and Bwlchgwyn became official stations contributing returns to the *Monthly Weather Report*. Under Ashmore's lead, Grove Park earned the reputation of providing one of the most complete day-to-day records of atmospheric optical phenomena in Britain. Bwlchgwyn has helped materially to fill a gap in our knowledge of Welsh upland climates.

Ashmore's popularity among his colleagues and pupils may have owed a little to his prowess as a sportsman. He was a noted skater and lawn tennis player. Roving the mountains of North Wales in all winds and weathers was one of his chief pastimes. For some years he undertook annual spring and summer visits to determine the amount of snow remaining in a deep gully on Carnedd Llewelyn where the winter accumulation occasionally survives until August and was at one time wrongly supposed to be quasi-permanent. He excelled as a ballroom dancer; not long before his death he was awarded the silver medal of the Imperial Society of Teachers of Dancing. He mastered the intricacies of Welsh poetry and could turn out a neat *englyn* in the strict ancient measures whenever occasion required.

Ashmore held the honorary post of Borough Meteorologist at Wrexham. As a lecturer on meteorological topics he became widely known in the Manchester-Liverpool area as well as in North Wales. He was an active member of Plaid Cymru, the Welsh Nationalist Party, and also of the Welsh Presbyterian Church, on whose behalf he taught regularly in a large Sunday school class. Almost the only relaxation he had permitted himself latterly was a week's attendance at the annual Eisteddfod Genedlaethol. He would not keep a radio receiving set in his rooms lest it might distract him from work. In his friends' belief his span of years might well have been longer if he had spared himself more and had not so often cut his nightly ration of sleep to three or four hours. But to watch the clock or to give of anything less than his best was not in Ashmore's nature.

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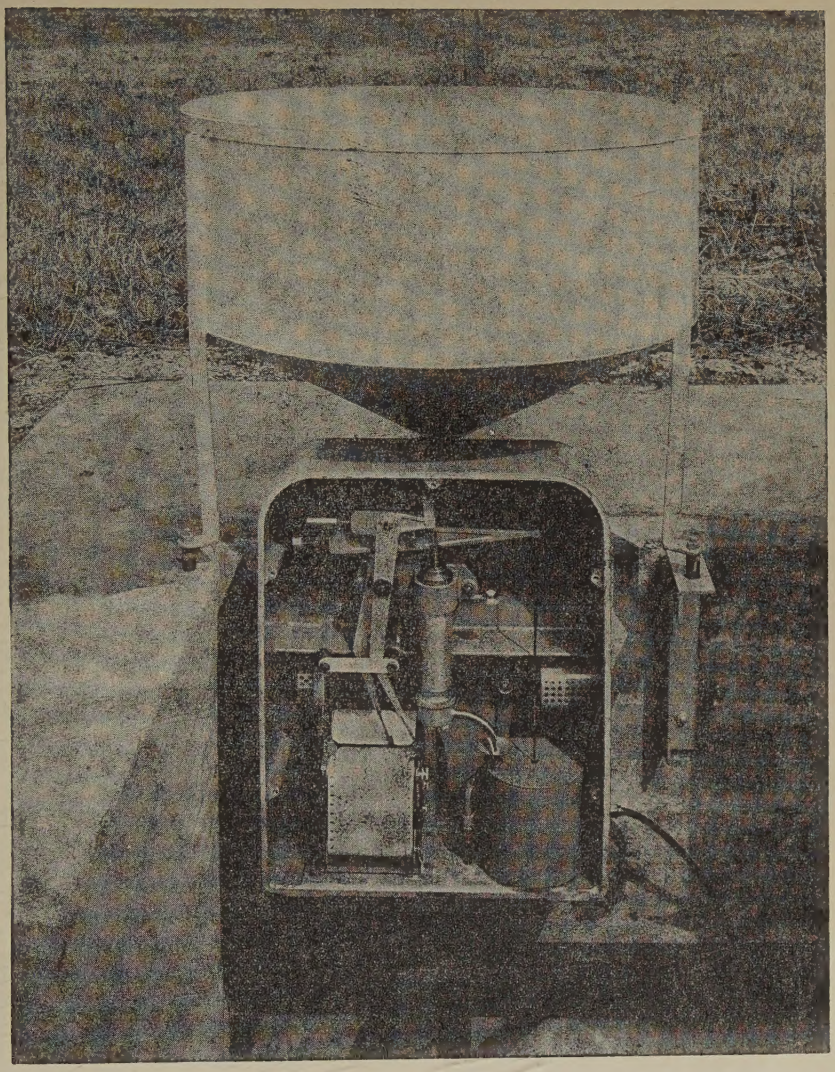

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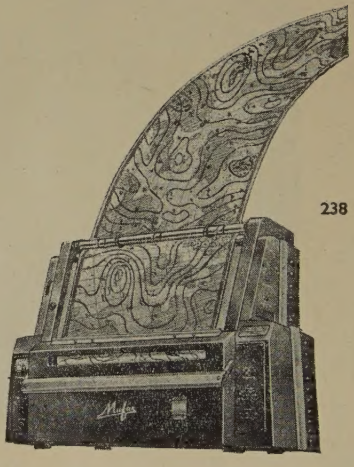

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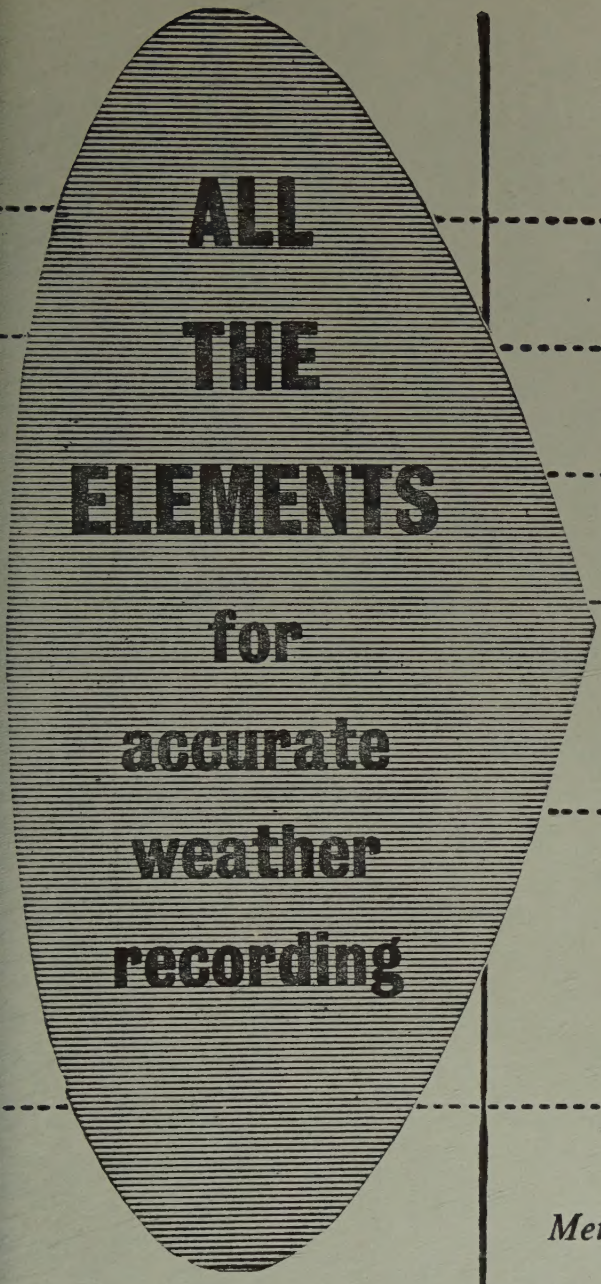

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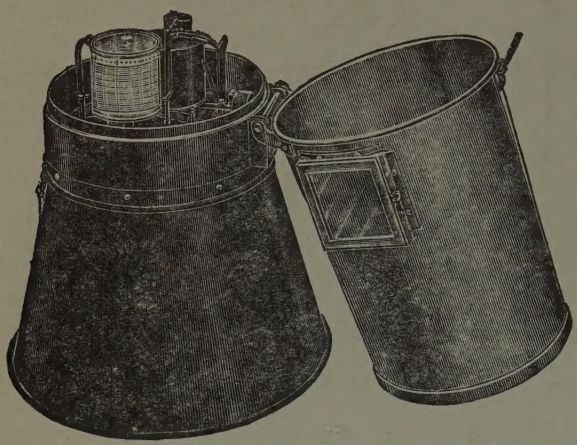

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